Managing Macroeconomic Fluctuations with Flexible Exchange Rate Targeting*

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Abstract

We show that a monetary policy rule that uses the exchange rate to stabilize the economy outperforms a Taylor rule in managing macroeconomics fluctuations and in achieving higher welfare. The differences between the rules are driven by: *(i)* the path of the nominal exchange rate and interest rate under each rule, and *(ii)* time variation in the risk premium, which leads to deviations from uncovered interest parity. These differences are larger in very open economies, more exposed to foreign shocks, and in which domestic and foreign goods are highly substitutable.

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1 Introduction

In the aftermath of the global financial crisis, unconventional monetary policies implemented by central banks in advanced economies resulted into an increase in liquidity in the financial system. This excess liquidity was channeled towards emerging economies as investors were "searching for yield". As a consequence, exchange rate volatility increased in these economies.

In small open economies, the exchange rate is an important element of the transmission of monetary policy (Svensson (2000)). Fluctuations in exchange rates have an effect on inflation and other economic variables and, thus, challenge the ability of monetary policy in stabilizing the economy. Because they are more subject to foreign shocks, central banks in emerging economies generally prefer to keep the exchange rate under tight control. In fact, many emerging economies today follow a quasi-managed floating exchange rate regime. Despite a large body of empirical research on managed floating, the theoretical literature discussing how monetary authorities deal with exchange rates has focused on corner solutions: either the currency rate is fixed by the central bank or the government, or it is left to be determined by market forces. First, a large number of papers evaluate the costs and benefits of fixed exchange rates (including Friedman (1953) and Flood & Rose (1995)). A second approach to incorporating the exchange rate into discussions of monetary policy is by augmenting a closed-economy Taylor rule with the rate of currency depreciation. Under this approach the interest rate reacts not only to inflation and the output gap but also to movements in the exchange rate. For example, De Paoli (2009) derives an optimal monetary policy rule within a DSGE model and shows that by putting some weight on real exchange rate fluctuations, a central bank can achieve improvements in social welfare. After Reinhart & Rogoff (2004) there has been growing interest in how the intermediate cases fare in terms of economic performance.

In this paper, we evaluate the properties of intermediate exchange rate regimes by considering an alternative class of policy rules where the central bank, instead of using the interest rate, adjusts the exchange rate in response to inflation and the output gap. The exchange rate is adjusted by the central bank in a manner similar to the use of the interest rate as an operating instrument. To evaluate the benefits of this rule relative to a standard interest rate rule, we build a DSGE model where the monetary authority adjusts the exchange rate in response to deviations of inflation from a target and fluctuations in the output gap.[1] [2]

[1]Benigno, Benigno & Ghironi (2007) show that it is possible to implement a fixed exchange rate regime with an interest rate rule for the follower country.

[2]The motivation for this type of rules comes from the Monetary Authority of Singapore (MAS). Since 1981, monetary policy has been implemented through flexible exchange rate targeting. See Parrado (2004), Khor, Lee, Robinson & Supaat (2007), McCallum (2007), and MAS (2012) offer detailed descriptions of the policy regime in Singapore.

Understanding the costs and benefits of an exchange rate policy rule within a fully specified model is not a trivial task. The immediate reaction is that if the model features an uncovered interest parity condition (UIP), then interest rate and exchange rate rules might generate similar outcomes. In our model, there are two reasons why the outcomes for the two rules differ. First, the actual implementation of the exchange rate rule is important. While the central bank technically can replicate any interest rate rule by moving the exchange rate today and announcing depreciation consistent with UIP, it is not the way that our rule operates. In our model, the exchange rate today is predetermined and the central bank announces the depreciation rate from time t to $t+1$. The underlying assumption is that the central bank can commit to a particular exchange rate next period. This implies for example, that the model may not feature the standard overshooting result as the currency rate both today and at $t+1$ are determined by the monetary authority. The simulations of our model suggest that this feature does generate differences between the two rules. A key factor for the exchange rate rule to be successful in reducing economic fluctuations is that the announcements of the central bank implementing the rule are credible. If the central bank is more than backed with foreign reserves and if it has built the credibility of maintaining low inflation, it will be less necessary to do continuous interventions, and the policy will be more successful. We abstract from credibility issues in the paper and assume that the exchange rate rule can be perfectly implemented.

Furthermore, these differences are amplified when UIP fails. Indeed, Alvarez, Atkeson & Kehoe (2007) argue forcefully that a key part of the impact of monetary policy on the economy goes through conditional variances of macroeconomic variables rather than conditional means. In terms of the UIP condition, their paper implies that the interest parity condition has a time-varying risk premium. Interest in time-varying risk premium has been growing in recent years. In the context of the interest parity condition, Verdelhan (2010) shows how consumption models with external habit formation can generate counter-cyclical risk premium that matches key stylized facts quite successfully. In our model, we adopt a similar approach by allowing external habit formation. To show the importance of the counter-cyclical risk premium, we report results for the first-order approximation, which wipes out the risk premium from UIP, and for the third-order approximation, which preserves time variation in the risk premium.[3]

We start by writing down a relatively standard New-Keynesian small open economy

[3] An alternative route for introducing risk premium in the UIP condition is by building in incomplete financial markets, as in Schmitt-Grohé & Uribe (2003), Turnovsky (1985), Benigno (2009) and De Paoli (2009). Deviations from UIP come from costs of adjusting holdings of foreign bonds. Limited participation models, are alternative incomplete market models that generate time-varying risk premia (Alvarez & Jermann (2001), Lustig & Van Nieuwerburgh (2005), Chien & Lustig (2009), and Alvarez, Atkeson & Kehoe (2009)). Alternatively, modeling recursive preferences and stochastic conditional variances, as in Backus, Foresi & Telmer (1995) and Backus, Gavazzoni, Telmer & Zin (2010), could generate large and variable risk premia in models with complete markets.

model as in Gali & Monacelli (2005) that we extend to include external habit in consumption, as in De Paoli & Zabczyk (2013). We then analyze the performance of the model under two different policy rules: a standard Taylor rule in which the monetary authority sets interest rates, and an alternative monetary rule in which the monetary authority sets the depreciation rate of the nominal exchange rate. We show that if UIP holds, these rules generate quantitatively responses to shocks. The Taylor rule implies overshooting of the exchange rate following a shock, generating a higher volatility of the exchange rate and other economic variables. We then introduce deviations from UIP. The goal is to analyze the performance of the two competing rules when the one-to-one relationship between exchange rates and interest rates breaks down. In this case, the differences between the two rules, in terms of the response of the economy to shocks, are amplified. The main reason is that the implementation of the monetary rule has an effect on the volatility of the risk premium through a precautionary saving motive. The Taylor rule features overshooting of the exchange rate, and it generates larger fluctuations of inflation and output gap, as the larger volatility of exchange rates increases the risk premium. The opposite is true for the exchange rate rule, as the monetary authority adjusts its path of appreciation to smooth economic fluctuations by generating a less-volatile exchange rate. In this regard, the exchange rate rule is also different from a peg, in which the monetary authority fixes the exchange rate to a specified value.[4]

After having exposed the mechanism driving the differences between the two rules, we evaluate quantitatively their performance both in terms of managing macroeconomic fluctuations and in terms of welfare. We begin by specifying a general rule in which the monetary authority reacts to fluctuations of inflation, output gap and exchange rate, with a certain degree of interest rate smoothing. We then compute, for each rule the implied volatility of key economic variables. We do this for: (i) a log-linearized version of our model which does not capture the existence of a time-varying risk premium, and (ii) a non-linear version of our model in which we log-linearize the demand and supply conditions, while taking a third-order approximation of the equations that depend on the risk premium directly. In that way, our non-linear model isolates the role of a time-varying risk premium and, hence, deviations for the UIP condition. We find that, for a wide range of plausible parameters in the monetary rules, the exchange rate rule outperforms the interest rate rule in terms of inflation and output growth. It also outperforms the peg. This is true for a large combination of parameter values in the monetary rules.

Finally, we compare the performance of the two rules in terms of welfare. Here, we

[4] Our results are consistent with those in Chow, Lim & McNelis (2014), who estimate a DSGE model for the Singapore economy under the two rules and find that the exchange rate rule outperforms the Taylor rule in reducing fluctuations of inflation. Different from their paper, we have a more general framework that can be applied to any small open economy. More importantly, our goal is to analyze the mechanisms behind the different performance of the two rules, especially those driven by the existence of a countercyclical risk premium that introduces deviation from UIP.

obtain parameter values of the two rules that maximize lifetime utility subject to the private sector's optimal behavior and given exogenous process of domestic productivity and foreign output shocks. More precisely, we evaluate lifetime utility for a wide range of parameter combinations in the two rules (in all the exercises we keep the smoothness parameter fixed at 0.85). We compute welfare by taking a third order approximation of the full model and of the utility function. In that way, we capture variations in the risk premium of the economy and we are able to evaluate how they impact the preferences of the central bank. Our welfare analysis is done using numerical approximations (see Collard & Juillard (2001), Schmitt-Grohé & Uribe (2004)).[5] We find that a policy rule in which the central bank adjusts the exchange rate to react to fluctuations of output and inflation is welfare improving with respect to a monetary rule in which the central bank uses the interest rate as its instrument. This is especially the case for very open economies and for economies in which the elasticity of substitution between domestic and foreign goods is large.

The rest of the paper proceeds as follows. Section 2 lays out the details of the model. Section 3 presents the mechanism of the exchange rate rule. Section 4 performs a quantitative analysis. Section 5 provides a summary of our key findings, some ideas for future research and conclusions.

2 The Model

Our model extends Gali & Monacelli (2005) by introducing a new policy rule based on using the exchange rate to stabilize the economy and adding external habit as in Campbell & Cochrane (1999), Jermann (1998), Verdelhan (2010) and De Paoli & Sondergaard (2009). Modeling assumptions are kept at a minimum to ensure that we can study the properties of the exchange rate rule without introducing too many confounding factors.

The basic framework is a dynamic general equilibrium model of a small open economy (Home) and the rest of the world (Foreign). It features complete international financial asset markets, monopolistic competition in production and sticky prices la Calvo. Prices are set in the producer's currency and the law of one price holds, hence there is complete exchange rate pass-through. However, due to home bias in consumption, there are devia-

[5] Several papers have derived analytically a welfare-based loss function of the small open economy.De Paoli (2009) uses the linear-quadratic approach developed by Sutherland (2002) and Benigno & Woodford (2006). We derive a second-order approximation of the utility function of the consumer in Appendix E, and find that the welfare-based loss function can be approximated by a linear-quadratic expression in domestic inflation, output gap, real exchange rate and the relative risk aversion. The last component reflects the presence of external habits in consumption, as they imply a time-varying coefficient of relative risk aversion. External habits introduce an additional trade-off in the economy that leads to over-consumption, as households do not internalize the effect of their consumption on the habit level of the economy (see De Paoli & Zabczyk (2013) and Leith, Moldovan & Rossi (2012) for a closed economy model).

tions from purchasing power parity and real exchange rate fluctuations. The economy is subject to a domestic productivity shock and a foreign output shock. We study the properties of the model under two alternative monetary policy rules: a conventional interest rate rule, in which the monetary authority uses the short term nominal interest rate as its instrument, and an exchange rate rule, in which the instrument is the exchange rate. In the remaining of the paper, we refer to these rules as IRR and ERR, respectively.

2.1 Households

In each country, there is a representative household who maximizes life-time expected utility. The utility function of the household in the Home country is given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(C_t - hX_t, N_t) \tag{1}$$

where N_t is hours of labor, X_t is the level of habits defined below, and C_t is a composite consumption index defined by:

$$C_t = \left[(1-\alpha)^{\frac{1}{\eta}}(C_{H,t})^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}}(C_{F,t})^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} \tag{2}$$

where $C_{H,t}$ denotes the consumption of domestic goods by the Home consumers, $C_{F,t}$ denotes the consumption of foreign goods by Home consumers, $\eta > 0$ is the elasticity of substitution between domestic and foreign goods, and $\alpha \in [0,1]$ is the degree of openness of the country. C_{Ht} and C_{Ft} are aggregates of intermediate products produced by Home and Foreign combined in the following way

$$C_{Ht} = \left[\int_0^1 C_{H,t}(i)^{\frac{\varepsilon-1}{\varepsilon}} di\right]^{\frac{\varepsilon}{\varepsilon-1}}; \quad C_{Ft} = \left[\int_0^1 C_{F,t}(i)^{\frac{\varepsilon-1}{\varepsilon}} di\right]^{\frac{\varepsilon}{\varepsilon-1}}, \tag{3}$$

with ε being the elasticity of substitution between varieties, which in turn are indexed by $i \in [0,1]$.

As in De Paoli & Sondergaard (2009) we assume that habits are external. We allow for flexibility in assessing the importance of habits by introducing in Equation (1) the parameter $h \in [0,1]$. When $h = 0$ the model collapses to the basic version of Gali & Monacelli (2005), while $h = 1$ corresponds to the modeling assumptions in Campbell & Cochrane (1999) and Verdelhan (2010). The evolution of habits follows an AR(1) process with accumulation of habits based on last-period consumption:

$$X_t = \delta X_{t-1} + (1-\delta)C_{t-1}, \tag{4}$$

Parameter $\delta \in [0,1]$ captures the degree of habit persistence. Again, this parameter allows us to consider various assumptions about habits with $\delta = 0$ corresponding to the assumptions in the earlier literature on habit-formation where habits are determined exclusively by the last-period consumption (e.g. Campbell (2003) and Jermann (1998)).

Consumers maximize (1) subject to the following budget constraint:

$$\int_0^1 P_{H,t}(i)C_{H,t}(i)di + \int_0^1 P_{F,t}(i)C_{F,t}(i)di + E_t\left\{\mathcal{M}_{t,t+1}B_{t+1}\right\} \leq B_t + W_tN_t \tag{5}$$

where $P_{H,t}(i)$ is the price of variety i produced at home, $P_{F,t}(i)$ is the price of variety i imported from Foreign (expressed in Home currency), $\mathcal{M}_{t,t+1}$ is the stochastic discount factor, B_{t+1} is the nominal payoff in period $t+1$ of the portfolio held at the end of period t, and W_t is the nominal wage.

The optimal allocation of expenditures within each variety gives the demand function for each product:

$$C_{H,t}(i) = \left(\frac{P_{H,t}(i)}{P_{Ht}}\right)^{-\varepsilon} C_{Ht}; \quad C_{F,t}(i) = \left(\frac{P_{F,t}(i)}{P_{Ft}}\right)^{-\varepsilon} C_{Ft}; \tag{6}$$

with $P_{H,t} = \left[\int_0^1 P_{H,t}(i)^{1-\varepsilon}di\right]^{\frac{1}{1-\varepsilon}}$ and $P_{F,t} = \left[\int_0^1 P_{F,t}(i)^{1-\varepsilon}di\right]^{\frac{1}{1-\varepsilon}}$ the price index of domestic and of imported goods (expressed in units of Home currency), respectively. From expression (6), $P_{Ht}C_{Ht} = \int_0^1 P_{H,t}(i)C_{H,t}(i)di$ and $P_{Ft}C_{Ft} = \int_0^1 P_{F,t}(i)C_{F,t}(i)di$.

The optimal allocation of expenditures between domestic and imported goods is:

$$C_{H,t} = (1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t; \quad C_{F,t} = \alpha\left(\frac{P_{F,t}}{P_t}\right)^{-\eta} C_t, \tag{7}$$

where $P_t = [(1-\alpha)P_{H,t}^{1-\eta} + \alpha P_{F,t}^{1-\eta}]^{\frac{1}{1-\eta}}$ is the consumer price index (CPI). From the previous equations, total consumption expenditures by the domestic households is $P_tC_t = P_{H,t}C_{H,t} + P_{F,t}C_{F,t}$. Therefore, we can re-write the budget constraint as

$$P_tC_t + E_t\left\{\mathcal{M}_{t,t+1}B_{t+1}\right\} \leq B_t + W_tN_t. \tag{8}$$

With per-period utility function of the following form

$$U(C_t, X_t, N_t) \equiv \frac{(C_t - hX_t)^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\gamma}}{1+\gamma}, \tag{9}$$

the first order conditions for the household's problem are

$$(C_t - hX_t)^{\sigma} N_t^{\gamma} = \frac{W_t}{P_t}, \tag{10}$$

$$\beta\left(\frac{C_{t+1} - hX_{t+1}}{C_t - hX_t}\right)^{-\sigma}\left(\frac{P_t}{P_{t+1}}\right) = \mathcal{M}_{t,t+1}. \tag{11}$$

Taking expectations on both sides, we have the Euler equation

$$R_t \cdot \mathbb{E}_t \left[\beta \left(\frac{C_{t+1} - hX_{t+1}}{C_t - hX_t} \right)^{-\sigma} \left(\frac{P_t}{P_{t+1}} \right) \right] = 1, \tag{12}$$

with $R_t \equiv 1/\mathbb{E}_t [\mathcal{M}_{t,t+1}]$ the gross return on a riskless one-period discount bond paying off one unit of domestic currency in $t+1$.

Below we elaborate on the need to use habit formation in this model but from the Euler equation it is already clear that the marginal utility of consumption increases when consumption goes down relative to the acquired level of habit. As we discuss below, this modeling approach generates a counter-cyclical coefficient of relative risk aversion leading to a counter-cyclical risk premium which drives a wedge between the interest rate differential and expected depreciation in the uncovered interest parity condition. Both Verdelhan (2010) and De Paoli & Sondergaard (2009) make this point quite forcefully.

2.2 Firms

We now characterize the supply side of the economy. In each country there is a continuum of monopolistically competitive firms, $i \in [0,1]$, that use labor to produce a differentiated good (each firm is associated with a different variety). Labor is the only factor of production, and we assume it to be immobile across countries.

Each firm i operates the linear technology

$$Y_t(i) = A_t N_t(i), \tag{13}$$

where domestic productivity follows and AR(1) process

$$\log(A_t) = \rho_A \log(A_{t-1}) + \log(U_{At}), \tag{14}$$

with σ_A the standard deviation of the innovation $\log(U_{At})$. Firms face downward-sloping demand from domestic and foreign households and maximize expected profits by setting the price of their varieties. Prices are set as in the Calvo model. A measure $1-\theta$ of randomly selected firms sets new prices every period. Since firms are identical, all resetting firms set the same new price, $\overline{P_{H,t}(i)} = \overline{P_{H,t}}$, such that we can drop the firm index i. It is well-known that firms optimally set

$$\frac{\overline{P_{H,t}}}{P_t} = \frac{\varepsilon}{\varepsilon - 1} \frac{H_t}{F_t}, \tag{15}$$

where H_t and F_t are auxiliary variables to express the pricing decision recursively and defined as

$$F_t \equiv \Lambda_t Y_t + \beta\theta \cdot \mathbb{E}_t \left(F_{t+1} \Pi_{t+1}^{\varepsilon-1} \right), \tag{16}$$

$$H_t \equiv \Lambda_t MC_t Y_t + \beta\theta \cdot \mathbb{E}_t \left(H_{t+1} \Pi_{t+1}^{\varepsilon} \right), \tag{17}$$

where $MC_t \equiv W_t / P_{H,t} A_t$ denotes real marginal costs (in units of the domestic goods) and $\Lambda_t \equiv (C_t - hX_t)^{-\sigma}$. In the Calvo model, domestic prices evolve following $P_{Ht} = \left((1-\theta) \overline{P_{H,t}}^{1-\varepsilon} + \theta P_{Ht-1}^{1-\varepsilon} \right)^{\frac{1}{1-\varepsilon}}$, which writes in real-terms as

$$\frac{P_{H,t}}{P_t} = \left((1-\theta) \left(\frac{\overline{P_{H,t}}}{P_t} \right)^{1-\varepsilon} + \theta \left(\frac{P_{H,t-1}}{P_{t-1}} \right)^{1-\varepsilon} \Pi_t^{\varepsilon-1} \right)^{\frac{1}{1-\varepsilon}}, \tag{18}$$

where $\Pi_t \equiv P_t / P_{t-1}$ is CPI inflation. We take the price of the home consumption bundle, P_t, as a numeraire and express all variables in real terms.

2.3 The Rest of the World

Because the foreign economy is exogenous to our small open economy, there is some flexibility in specifying the behavior of the foreign variables. We assume foreign output follows an AR(1) process,

$$\log (Y_t^*) = \rho_{Y^*} \log \left(Y_{t-1}^* \right) + \log (U_{Y^*t}), \tag{19}$$

with σ_{Y^*} the standard deviation of innovations $\log (U_{Y^*t})$. With external habits in consumption in the rest of the world, the foreign household's inter-temporal first order condition is

$$\mathcal{M}_{t,t+1}^* = \beta \left(\frac{C_{t+1}^* - hX_{t+1}^*}{C_t^* - hX_t^*} \right)^{-\sigma} \left(\frac{P_t^*}{P_{t+1}^*} \right) \tag{20}$$

with $\mathcal{M}_{t,t+1}^*$ the foreign economy's stochastic discount factor, C_t^* the foreign consumption-bundle, X_t^* the stock of habits accumulating as

$$X_t^* = \delta X_{t-1}^* + (1-\delta) C_t^*, \tag{21}$$

and P_t^* the foreign CPI. Taking expectations of Equation (20) yields the foreign economy's Euler Equation and the foreign interest rate as $R_t^* \equiv 1 / \mathbb{E}_t \left[\mathcal{M}_{t,t+1}^* \right]$. For simplicity, we assume that the foreign CPI is fully stabilized $P_t^* = 1$.[6] Finally, the foreign household's

[6]Full stabilization of the foreign CPI implies that the foreign interest rate is directly pinned down by the foreign economy's Euler equation. Alternatively, we could allow the foreign consumer price index to fluctuate and introduce an interest rate monetary policy rule to determine the foreign interest rate as a function of foreign inflation. Our results are robust to this alternative specification, since the foreign economy is fully exogenous to our small open economy.

intratemporal optimization yields demand for the domestic good as

$$C_{H,t}^{*} = \alpha \left(\frac{P_{H,t}^{*}}{P_{t}^{*}} \right)^{-\eta} C_{t}^{*}, \tag{22}$$

where $P_{H,t}^{*}$ denotes the price of domestic goods in the rest of the world.

2.4 Two Monetary Policy Rules: IRR versus ERR

We consider two alternative monetary policy rules. First, we analyze the model under a standard Taylor rule, in which the monetary authority sets the nominal interest rate to smooth fluctuations in CPI inflation, output, and the nominal exchange rate depreciations,

$$\frac{R_t}{\bar{R}} = \left(\frac{R_{t-1}}{\bar{R}} \right)^{\rho} \left[\left(\frac{\Pi_t}{\overline{\Pi}} \right)^{\phi_\Pi} \left(\frac{Y_t}{\overline{Y}} \right)^{\phi_Y} \left(\frac{\Delta \mathcal{E}_t}{\overline{\Delta \mathcal{E}_t}} \right)^{\phi_E} \right]^{1-\rho} \tag{23}$$

where $\rho \in (0,1)$ is the degree of interest rate smoothing.

Second, we consider a monetary policy rule in which the central bank adjusts the path of the nominal exchange rate to stabilize CPI inflation and output. The exchange rate depreciation policy, $\Delta \mathcal{E}_{t+1}^{*}$, is adjusted in response to deviations of these variables from their targets,

$$\frac{\Delta \mathcal{E}_{t+1}^{*}}{\overline{\Delta \mathcal{E}_{t+1}}} = \left(\frac{\Pi_{t+1}}{\overline{\Pi}} \right)^{-\phi_\Pi} \left(\frac{Y_{t+1}}{\overline{Y}} \right)^{-\phi_Y} \tag{24}$$

with $\overline{\Delta \mathcal{E}_{t+1}}$ the depreciation required to reach the long-run equilibrium nominal exchange rate.[7] We assume that there is some smoothing in the way the nominal exchange rate adjusts to its target level

$$\Delta \mathcal{E}_{t+1} = \Delta \mathcal{E}_t^{\rho} \left[\Delta \mathcal{E}_{t+1}^{*} \right]^{(1-\rho)} \tag{25}$$

When inflation and output are high, the central bank announces a path of appreciation of the exchange rate that is given by $\Delta \mathcal{E}_t^{*}$ and this determines the evolution of the nominal exchange rate. Note that this rule corresponds to a managed float and it is between a completely fixed exchange rate regime in which $\Delta \mathcal{E}_{t+1}$ and a flexible exchange rate regime, that would correspond to the central bank using as an instrument the nominal interest rate, letting the exchange rate fluctuations be driven by market forces.[8]

2.5 Market Clearing Conditions

Here, we describe the markets clearing conditions in the goods market and in the financial assets markets.

[7]We follow the convention that an increase in the exchange rate implies depreciation of the domestic currency, as in Parrado (2004).

[8]The exchange rate rule and its properties have been documented and estimated for the case of Singapore by Parrado (2004) and Khor, Lee, Robinson & Supaat (2007).

Goods Market

Since the Home economy is small, its demand for foreign good is small as well, and the goods market clearing condition for the foreign good can be expressed as

$$Y_t^* = C_t^*. \tag{26}$$

Demand for the domestic good is

$$Y_t^d \equiv C_{H,t} + C_{H,t}^*$$

Substituting equations (7) and (22), we have

$$Y_t = \left(\frac{P_{H,t}}{P_t}\right)^{-\eta} \left[(1-\alpha)C_t + \alpha Q_t^\eta C_t^*\right], \tag{27}$$

where we used the fact that the law of one price holds $P_{H,t} = \mathcal{E}_t P_{H,t}^*$ (with $\mathcal{E}_t$ the nominal exchange rate denoted in units of domestic currency per unit of foreign currency as in Gali & Monacelli (2005)) and the definition of the real exchange rate as the relative price of the foreign consumption bundle in terms of the domestic consumption bundle, $Q_t \equiv \mathcal{E}_t P_t^* / P_t$.

International Risk Sharing and the UIP Condition

We assume that financial markets are complete and all contigent claims are traded internationally. Therefore, stochastic discount factors at Home and Foreign must be equalized in equilibrium in terms of the domestic currency[9],

$$\mathcal{M}_{t,t+1}^* = \Delta\mathcal{E}_{t+1} \cdot \mathcal{M}_{t,t+1}, \tag{28}$$

where $\Delta\mathcal{E}_{t+1}$ denotes the nominal depreciation of the domestic currency. Together with the Euler equations, we obtain that marginal rates of intertemporal substitution in real terms are equalized between Home and Foreign and risk is perfectly shared in all states of the world. That is,

Equation (28) also holds in expectation implying an equilibrium relation between domestic and foreign interest rates, nominal exchange rate depreciation, and the domestic stochastic discount factor,

$$\mathbb{E}_t \left[\mathcal{M}_{t,t+1} \left(R_t - R_t^* \cdot \Delta\mathcal{E}_{t+1}\right)\right] = 0. \tag{29}$$

[9] The stochastic discount factor $\mathcal{M}_{t,t+1}$ ($\mathcal{M}_{t,t+1}^*$) can also be interpreted as the price of an Arrow-Debreu security paying of one unit of domestic (foreign) currency in any given state of the world in $t+1$. If all securities are traded internationally, no-arbitrage implies that returns to one unit of the domestic currency are equalized, i.e. $1/\mathcal{M}_{t,t+1} = 1/\mathcal{E}_t \cdot 1/\mathcal{M}_{t,t+1}^* \cdot \mathcal{E}_{t+1}$.

Log-linearization of Equation (29) around a perfect foresight steady state yields the standard uncovered interest parity condition (small letters with a hat denote log deviations from steady state),

$$\hat{r}_t^* = \hat{r}_t + \mathbb{E}_t\left[-\Delta \hat{e}_{t+1}\right], \tag{30}$$

Alvarez, Atkeson & Kehoe (2007) argue that assumptions leading to this simplified interest parity condition imply dynamics that are inconsistent with the data. Under assumptions of conditional log-normality of the stochastic discount factor, a time-varying risk premium emerges, as shown by Backus, Gavazzoni, Telmer & Zin (2010). Alternatively, a higher-order approximation of the Euler equation can also generate time-varying risk premium.[10].

Habit-based preferences, as specified in Equations (4) and (9), give rise to counter-cyclical risk aversion[11]: When consumption is close to the habit level in period t, the household is more averse towards risk in period $t+1$. Such counter-cyclical risk aversion is reflected in the household's pricing kernel, $\mathcal{M}_{t,t+1}$, and therefore affects the premium required by domestic households for holding foreign bonds. Foreign bonds carry exchange rate risk from the viewpoint of the domestic investor since they pay in foreign currency. For example, if the nominal exchange rate appreciates in states of low future consumption, a domestic investor demands a higher return on foreign bonds to compensate risk exposure.

An analytical expression of the exchange rate risk premium in our model is beyond the scope of the paper. However, to gain intuition on the components that drive the exchange rate risk premium, we follow Backus, Gavazzoni, Telmer & Zin (2010), who assume that the stochastic discount factors are jointly lognormal. Hence, Equation (29) becomes exactly[12]

$$\hat{r}_t^* = \hat{r}_t + \mathbb{E}_t\left[-\Delta \hat{e}_{t+1}\right] + \underbrace{\mathbb{C}ov_t\left[\frac{\hat{m}_{t,t+1} + \hat{m}_{t,t+1}^*}{2}, -\Delta \hat{e}_{t+1}\right]}_{\equiv fxp_t}, \tag{31}$$

where fxp_t denotes the exchange rate risk premium. Equation (31) shows that the exchange rate risk premium is a function of the conditional (on time t) covariance of the average pricing kernel with the nominal appreciation of the domestic currency.

The return on foreign bonds carries a risk premium for two reasons: First, domestic investors require a higher return on foreign bonds if the domestic currency systematically appreciates in states of low consumption and/or inflation, $\mathbb{C}ov_t\left[\hat{m}_{t,t+1}, -\Delta \hat{e}_{t+1}\right] > 0$.

[10]At a first order approximation of the model, there is no risk premium; at a second-order approximation, there is a risk premium but it is constant.

[11]As shown De Paoli & Sondergaard (2009) the coefficient of relative risk aversion can be defined as $\sigma_t \equiv -C_t U_{CC}/U_C = \sigma/C_t - hX_t$.

[12]As mentioned in De Paoli & Zabczyk (2013), this expression also holds up to the second order without any distributional assumptions on the pricing kernels.

This is because foreign currency investments pay-off less in states where the marginal utility of currency is high. Second, foreign investors require compensation for holding foreign bonds if the home currency appreciates in states where foreign consumption is low, $\mathbb{C}ov_t\left[\hat{m}^*_{t,t+1}, -\Delta\hat{e}_{t+1}\right] > 0$.[13]

Equation (31) illustrates that domestic financial conditions are impacted by the dynamics of the exchange rate risk premium, which provides new opportunities for monetary policy. A small open economy with fully integrated financial markets that takes world interest rates as given may be able to exploit variations in time-varying risk premia to stabilize domestic macroeconomic fluctuations. Through managing the conditional covariance structure of the economy (i.e. fxp_t), monetary policy could shield domestic interest rates from foreign interest rate fluctuations while keeping the nominal exchange rate relatively stable.

3 The Mechanism

Here, we provide an analysis of how the model works under an exchange rate rule in two ways. First, we describe, for our exchange rate rule, the transmission channels of monetary policy that have been studied extensively under interest rate rules. Second, we analyze the main differences between the two rules considered in the paper in terms of managing macroeconomic fluctuations.

3.1 ERR Transmission Channels

To illustrate the main transmission channels of the ERR, we assume that the monetary authority follows a simplified ERR that targets only CPI inflation, with some degree of interest rate smoothing. That is

$$\Delta\hat{e}_t = \rho\Delta\hat{e}_{t-1} - (1-\rho)\phi_\Pi\hat{\pi}_t. \tag{32}$$

Consider a negative domestic productivity shock. Inflation increases, as there is an excess demand for home goods in the domestic economy and the monetary authority reacts by announcing a path of appreciation of the nominal exchange rate in the following way. In period t, the increase in inflation following the shock, $d\widehat{\pi}_t > 0$, translates into an appreciation of $d\Delta\hat{e}_t = -(1-\rho)\phi_\Pi d\hat{\pi}_t$ in period t, $d\Delta\hat{e}_{t+1} = -(1-\rho)\phi_\Pi\left(\rho d\hat{\pi}_t + d\hat{\pi}_{t+1}\right)$ in period $t+1$, $d\Delta\hat{e}_{t+2} = -(1-\rho)\phi_\Pi\left(\rho^2 d\hat{\pi}_t + \rho d\hat{\pi}_{t+1} + d\hat{\pi}_{t+2}\right)$ in period $t+2$, and so on. Therefore, changes in current inflation today transmit to the appreciation in the nominal

[13]Under conditional log-normality, the foreign exchange rate risk premium can also be expressed as $fxp_t = \frac{1}{2}\left[\mathbb{V}ar_t\left[\hat{m}_{t+1}\right] - \mathbb{V}ar_t\left[\hat{m}^*_{t+1}\right]\right]$. In good times in the domestic economy, the variance of the domestic discount factor is low, relative to the variance of the foreign discount factor, and hence the risk premium that domestic investors demand to hold foreign bonds is lower.

exchange rate over time.

We argue that this appreciation policy transmits to economic decisions through three distinct channels: *(i)* intra-temporal demand re-balancing, *(ii)* inter-temporal consumption-saving decisions, and *(iii)* endogenous exchange rate risk premium dynamics. While the first two operate already at a first-order approximation of the model, the third is only active at third- or higher-order approximations.

The first two channels are related to the dual role of the exchange rate as price of financial assets *and* goods (Corsetti, Dedola & Leduc (2010)). By managing the exchange rate when there are fluctuations of inflation and output, the monetary policy directly influences both the goods and the financial markets. As a result, households revise their consumption plans because of inter-temporal consumption-saving motives – similarly as it occurs under conventional interest rate rules – as well as because of the intra-temporal trade-off between consuming domestic or foreign goods.[14] We explain each channel in detail next.

Intra-Temporal Demand Re-balancing On the goods markets, the immediate nominal appreciation implemented by ERR deteriorates the terms of trade at Home, as domestic goods become more expansive, relative to foreign goods. The appreciation of the exchanges rate induces rebalancing of domestic demand foreign goods and this effect is sufficient to clear the excess demand caused by the shock. As a result, prices adjust less in order to equalize demand and supply. and inflation is more stable in equilibrium.

The intra-temporal channel of demand rebalancing can be illustrated by total differentiating the demand for home goods in the log-linerized model (see Appendix D). Noting $\hat{y}_t^d$ the log-deviation of demand for home goods from steady state,

$$d\hat{y}_t^d = \underbrace{-\frac{\eta\alpha(2-\alpha)}{1-\alpha}\left(1+(1-\rho)\phi_\Pi\right)}_{\textit{Demand Rebalancing}} \cdot d\hat{\pi}_t + (1-\alpha)d\hat{c}_t + \alpha d\hat{c}_t^*. \tag{33}$$

The effect of the ERR on domestic demand is stronger, the more domestic goods are substitutes of foreign goods (η), the more open the economy (α), and the stronger the reaction of monetary policy to CPI inflation ($(1-\rho)\phi_\Pi$).

Inter-Temporal Substitution On the financial markets, the nominal appreciation following the shock decreases the expected return on foreign bond investments in domestic currency, $\hat{r}_t^* + \mathbb{E}\left[\Delta\hat{e}_{t+1}\right]$. The ERR effectively turns foreign bonds into CPI inflation-indexed bonds by tying the nominal exchange rate to domestic monetary policy targets. If CPI inflation increases, returns on foreign bond investments decrease, and vice versa.

[14] Under an interest rate rule, on the other hand, the central bank only directly intervenes on the financial markets through setting the risk-free interest rate. Intra-temporal shifts in consumption remain solely determined by market forces.

The lower return on foreign investments induces immediate capital inflows from the rest of the world which boost domestic asset prices and push down domestic interest rates which are determined in the financial market equilibrium (Equation 30).

In contrast to interest rate rules, however, households have under the ERR an incentive to save *more* and consume less when domestic interest rates decrease. The reason is that under the ERR the domestic CPI level is stationary: forward-looking households anticipate that higher prices today imply lower prices in the future.[15] As under an exchange rate peg, households prefer to shift – in response to current inflation – consumption into future periods, where goods are expected to become cheaper again.[16] By appreciating the nominal currency in states of high inflation, the ERR induces capital inflows pushing down the domestic interest rate. Households thus need to save more if interest rates decrease in order to shift consumption to the future.

To illustrate the inter-temporal substitution channel under the ERR (see Appendix D), we combine the Euler equations (12), (20) with the log-linearized UIP condition in equation (30) and the exogenous process of foreign output in equation (19), and obtain

$$d\hat{c}_t = \underbrace{-(1-h)\sigma^{-1}\left(1+(1-\rho)\phi_{\Pi}\right)}_{Consumption-Saving} \cdot d\hat{\pi}_t + d\hat{c}_t^*. \tag{34}$$

The consumption-saving channel is stronger, the lower the degree of habit formation (h), the more households are willing to smooth consumption inter-temporally ($1/\sigma$), and the stronger the reaction of the monetary authority to CPI inflation ($(1-\rho)\phi_{\Pi}$).

FX Risk Premium and Precautionary Savings The exchange rate risk premium provides an additional degree of freedom in the trilemma between independent domestic monetary policy, fixed exchange rate, and free capital flows (Mundell (1963) and Fleming (1962)). For example, for given foreign interest rate and exchange rate expectations, an increase in the equilibrium exchange rate risk premium pushes down domestic interest rates.

Under the simplified ERR, the risk premium in equation (31) is a function of the conditional covariance between stochastic discount factors and CPI inflation.[17] We follow the same reasoning as in Section 2.5.

[15] Gali & Monacelli (2005) show that prices are stationary under an exchange rate peg. We verify this to be the case under our ERR as well.

[16] This is different to the inter-temporal substitution channel operating under interest rate rules. Under IRR, domestic CPI are not stationary, and forward-looking households do not expect CPI prices to return to their initial level in the future. Rather, inflation today implies expected inflation tomorrow, consumption only decreases in response to current inflation if the central bank increases interest rates sufficiently – i.e. more than one-for-one (Taylor Principle) – such that saving more and consuming less in the current period becomes beneficial.

[17] Plugging the ERR into the expression for the exchange rate premium shows that under ERR: $fxp_t = \mathbb{C}ov_t\left[\left(\hat{m}_{t,t+1}+\hat{m}^*_{t,t+1}\right)/2, (1-\rho)\phi_{\Pi}\hat{\pi}_{t+1}\right]$.

Forward-looking households anticipate that higher CPI inflation induces a nominal currency appreciation policy. Therefore, the return on foreign bond investments is low in high inflation states (stochastic discount factor is low), and high in low inflation states (stochastic discount factor is high). If high inflation states are associated with high consumption (e.g. after a positive foreign output shock), the stochastic discount factor is unambiguously low in high inflation states and high in low inflation states.[18] Households accept a lower payoff on foreign bonds relatively to domestic bonds because of the insurance benefits of foreign bonds. The exchange rate is on average negative ($fxp_t < 0$).[19]

As shown by De Paoli & Zabczyk (2013), the dynamics of the risk premium under external habits are determined by fluctuations in *(i)* risk aversion and *(ii)* consumption prospects. If risk aversion is high or consumption prospects are poor, households ask more compensation for exchange rate risk and the risk premium increases. Risk aversion is counter-cyclical and consumption prospects are pro-cyclical if habits (δ) and shocks (ρ_A, ρ_{Y^*}) are persistent enough[20] or future prices are expected to decrease sufficiently with increasing consumption (and vice versa)[21]. The risk premium decreaeses if prospects improve or only deteriorate by little with increasing consumption. Households become less precautionary (the domestic discount factor is less volatile), save less in riskless domestic bonds, and accepts a lower excess return on foreign (risky) bonds. Vice versa, the risk premium increases with foreign consumption if prospects in the rest of the world improve or only deteriorate by little. In this case, foreigners become less precautionary (the volatililty of the foreign discount factor goes down) and save less in the foreign (riskless from the viewpoint of the foreign investor) bond. Foreigners accept a lower excess return on domestic bonds.

These dynamics can stabilize domestic prices: if inflation increases under the ERR, the central bank curbs demand by implementing a path of nominal currency appreciation. Through inter-temporal substitution households save more and consume less. If the risk premium is counter-cyclical – increases with lower consumption – domestic households become more precautionary and save even more and consume less. The risk premium

[18] If, on the other hand, high inflation states are associated with low consumption (e.g. after a negative domestic productivity shock), the stochastic discount factor may increase in high inflation states. In this case, investors demand a positive risk premium ($fxp_t > 0$) on foreign bond investments on average. We find that in our small open economy the foreign output shock dominates the conditional covariance of consumption with inflation.

[19] This allows the small open economy, for example, to sustain higher domestic interest rates for any given foreign interest rate and currency appreciation paths (or vice versa to sustain an appreciated currency on average). This is the terms of trade externality (see for example De Paoli & Sondergaard (2009)). The small open economy has an incentive to appreciate its real exchange rate on average to increase consumption possibilities (of foreign goods) without increasing disutility from labor.

[20] For example, if a positive shock is expected to persist, consumption prospects improve, whereas they deteriorate if the shock is rather transitory.

[21] For example, consumption prospects may increase if prices go down sufficiently for several periods after a positive domestic productivity shock. This inflation channel is absent in both De Paoli & Zabczyk (2013) and De Paoli & Sondergaard (2009) who analyze risk premium dynamics in flexible price economies.

thus re-inforces the fall in consumption. Domestic prices need to go up less to clear excess demand.

3.2 Implications of the IRR and ERR for Business Cycle Dynamics

The two monetary rules have different implications for business cycle fluctuations. These differences are amplified at a third-order approximation, due to the presence of an endogenous and time-varying risk premium that breaks down the UIP condition.[22]

To understand how the two rules imply different business cycle dynamics, we first log-linearize equations (23) and (24). From equation (23), we have

$$\hat{r}_t = \rho \hat{r}_{t-1} + (1-\rho)\left[\phi_\Pi \hat{\pi}_t + \phi_Y \hat{y}_t\right]. \tag{35}$$

Under a Taylor rule, the central bank increases the nominal interest rate when inflation or ouput are higher than their targets. Instead, if inflation or output are low, the central bank stimulates the economy by lowering the nominal interest rate.

Similarly, if we put together equations (24) and (25), and we log-linearize the expression, we have

$$\Delta \hat{e}_t = \rho \Delta \hat{e}_{t-1} - (1-\rho)\left[\phi_\Pi \hat{\pi}_t + \phi_Y \hat{y}_t\right]. \tag{36}$$

Under an exchange rate rule, the central bank stimulates the economy by depreciating the currency when inflation or output is low. In this case, the nominal exchange rate is depreciated (goes up) to increase demand from the rest of the world. Notice that, unlike in the Taylor rule, when lower inflation or output lead to a lower interest rate, the gradual depreciation under the exchange rate rule leads to an increase in the interest rate, through the UIP condition.[23] When UIP does not hold, there is an additional effect on the nominal interest rate, which is driven by a precautionary saving motif.

To illustrate the different implications for business cycle dynamics, consider a positive domestic productivity shock. A central bank that follows an interest rate rule decreases the nominal interest rate (see equation (35)) inducing capital outflows which depreciate the domestic currency. When UIP holds, non-arbitrage determines a future appreciation of the domestic currency, which leads to an overshooting of the nominal exchange rate. Under an exchange rate rule, however, the overshooting does not happen. The reason is that, after the shock, the central bank reacts by announcing a slow depreciation of the

[22] At a first order approximation of the model, there is no risk premium; at a second-order approximation, there is a risk premium but it is constant

[23] This is somewhat counterintuitive as "fighting inflation" implies a lower interest rate. The communication from the Monetary Authority of Singapore is quite clear in terms of using appreciation of the currency to lower inflation. Anecdotally, one can see the decline in interest rates, e.g. the announced appreciation of the Singapore dollar in October 2007, led to a decline in the one-month rates from 2.5% to 2.38% within a month and eventually to 1.13% before the following announcement in April 2008.

currency (see equation (36)). As forward-looking consumers expect that the currency will continue to depreciate, there is an excess supply of domestic bonds, which lowers the price of these bonds, hence increasing the domestic interest rate. When UIP holds, the increase in the nominal interest rate equals the expected future depreciation (see equation (30)). Because there is no overshooting, this increase is lower than the decrease in the domestic interest rate under the Taylor rule, and hence the exchange rate rule generates less fluctuations of domestic variables. Some degree of exchange rate smoothing in the rule is key to avoid overshooting under the exchange rate rule. The two rules imply differences in business cycle fluctuations due to the different response of the exchange rate and the nominal interest rate.

External habits introduce deviations of the UIP condition at higher order approximations through an endogenous and time-varying risk premium. This leads to additional differences in business cycle fluctuations between the interest rate rule and the exchange rate rule.

A positive domestic productivity shock generates excess supply of domestic goods and firms set lower prices. The central bank stimulates demand to stabilize the economy. Under the IRR, the central bank decreases the interest rate which leads to capital outflows and a large currency depreciation and a future appreciation (overshooting). While under the ERR, it implements a depreciation path of the currency. This also leads to capital outflows since the expected depreciation increases the return on foreign bond investments and domestic interest rates increase. Under both rules, households increase consumption and with a counter-cyclical risk premium decrease their pre-cautionary savings. This amplifies the consumption increase under both rules. This effect is weaker under the ERR because the exchange rate is less volatile (no overshooting) and so the risk premium decreases less.

4 Quantitative Analysis

We calibrate the model and perform a quantitative analysis to compare the macroeconomic performance of the ERR and IRR. We do this for: *(i)* a first order approximation of the model, which does not capture the existence of a risk premium, and *(ii)* for a third order approximation of the model that captures a time-varying risk premium. Here, as in De Paoli & Sondergaard (2009), we use the log-linear version of the demand and supply conditions, while taking a third-order approximation of the equations that directly depend on the risk premium. In that way, our non-linear model isolates the role of a time-varying risk premium. We call this the *hybrid* model. We use Dynare for our numerical exercises. Moments are based on simulations of 10,000 periods and we use pruning to avoid non-stationarity of simulations at third-order approximation.

We start by analyzing impulse responses of one-standard-deviation shocks to domestic productivity and foreign output to evaluate the business cycle properties of the different rules and shed light on the mechanism driving the differences. At a first order approximation of the model, differences between the two rules depend on the actual implementation: the fact that the central bank announces a gradual path of appreciation or depreciation of the currency and the direct intervention of the exchange rate rule in the goods market. At a third-order approximation, the performance of the policy rules will additionally depend on how they affect the dynamics of the risk premium (see Van Binsbergen, Fernandez-Villaverde, Koijen & Rubio-Ramirez (2012)).

In Section 4.3, we conduct a more formal quantitative analysis in which, first, we evaluate differences between the two rules in terms of second moments. We consider generalized monetary rules in which the central bank reacts to deviations of inflation, output, and exchange rates. We also allow for a wide range of parameter values in both rules and check whether there exists any combination of the parameters for which the two rules deliver business cycle volatility. Second, we compare the two rules in terms of welfare.[24]

4.1 Baseline Calibration

The calibrated parameters are reported in Table A.1. We follow closely the parametrization of De Paoli & Sondergaard (2009) and Gali & Monacelli (2005). The model is calibrated at a quarterly frequency. The parameters of habit persistence are set to $h = 0.85$ and $\delta = 0.99$. The elasticity of substitution across intermediate goods, ε equals 6, and between domestic and foreign goods is $\eta = 1$. We set the inverse of Frisch elasticity of labour supply, γ equal to $3/1-h$.[25] The degree of openness, α, is set to 0.08 (see Lubik & Schorfheide (2007)). The discount factor is set at $\beta = 0.99$, which implies a steady-state interest rate of 4% in a quarterly model. We assume the degree of price stickiness to be $\theta = 0.67$, which is consistent with the average period of price adjustment of 3 quarters, and the inverse of the intertemporal elasticity of substitution to be $\sigma = 5$, which is within the range found by the empirical literature of $[2, 10]$.

As in Gali & Monacelli (2005), domestic productivity is assumed to have a standard deviation of $\sigma_A = 0.71\%$, and foreign productivity shock has a standard deviation of $\sigma_{Y^*} = 0.78\%$. The shocks are assumed to be positively correlated with correlation $\sigma_{A,Y^*} = 0.3$.

4.2 Business Cycle Dynamics: Impulse Response Functions

For simplicity, and to illustrate our mechanism, we consider simplified rules in which the monetary authority adjusts either the nominal interest rate or the nominal exchange rate

[24]In the appendix, we also derive the welfare-based loss function and characterize the Ramsey constraint optimal policy.

[25]We follow De Paoli & Sondergaard (2009) who argue that under external habits, consumption would be too smooth relative to the data for lower values.

to react to fluctuations of inflation only, with a certain degree of smoothing. For the Taylor rule, we set $\phi_\Pi = 1.5$, $\phi_Y = 0$ and $\rho = 0.85$ as in Lubik & Schorfheide (2007). For the exchange rate rule, there is not a clear value for ϕ_Π that allows us to compare the two rules exactly. Here we consider $\phi_\Pi = 1$, a value that is consistent with estimates found in the literature for the case of Singapore (see Parrado (2004)). We assume the same degree of smoothing in both rules.

4.2.1 First-Order: Uncovered Interest Parity Holds

After a positive domestic productivity shock (Figure B.1), both output and consumption increase. Domestic inflation decreases because the economy is now more productive and there is excess demand for home goods. The central bank stimulates the economy differently under the two rules. If the central bank follows a Taylor rule, it sets a lower interest rate and the domestic currency depreciates. The initial depreciation is followed by a future appreciation, since UIP holds. There is overshooting of the nominal exchange rate. CPI inflation decreases because the initial decrease in domestic inflation dominates the depreciation of the currency. If, instead, the central bank follows the ERR, after the positive domestic productivity shock, the central bank reacts to the fall in inflation by announcing a gradual depreciation of the exchange rate. In this case, the nominal interest rate increases, since, as households expect a future depreciation of the currency, there is an excess supply of domestic bonds. Therefore, interest rates move in opposite directions under the two rules and the exchange rate is more stable under the ERR because there is no overshooting.

After a positive foreign output shock (Figure B.2), differences in business cycle dynamics caused by the two monetary rules are even more pronounced. Common to both rules, the shock has the two immediate effects that foreign interest rates fall and there is excess demand by the rest of the world for home goods. The capital inflows induced by lower foreign interest rates, however, have a different effect under each rule: while under the IRR it is the exchange rate that is determined by market forces (and the interest rate that is set by the central bank), it is the interest rate that is free-floating under the ERR and the exchange rate is determined as a function of domestic monetary policy targets.

Under the IRR, capital inflows immediately and large appreciate the nominal exchange rate (overshooting followed by expected depreciations through UIP) and deteriorate the small open economy's terms of trade. This leads to large rebalancing of aggregate demand away from home goods and towards foreign goods. As a result there is excess supply for home goods, firms decrease domestic prices and produce less. The central bank *stimulates* the economy by decreasing the domestic interest rate in response to deflation which boosts consumption today and cushions the nominal currency appreciation. Under the ERR, on the other hand, capital inflows directly pass-through and push down domestic interest rates. Households save less and consume more which amplifies the initial excess demand for home goods by foreigners. Firms increase domestic prices. The central banks aims to *curb* demand and announces a path of currency appreciation in response to inflation. This deteriorates the small open economy's terms of trade which rebalances aggregate demand towards foreign goods and decreases consumption through intertemporal substitution.

After a positive foreign output shock, the ERR, thus, stabilizes much more the exchange rate by avoiding overshooting. This shields the domestic economy from too strong deteriorations in terms of trade and leads to modest inflation – in contrast to large deterioration in terms of trade and a slump in demand and, consequently, output under IRR.

Our impulse response analysis in log-linear model shows that there exist qualitative differences in business cycle dynamics driven by the implementation of the monetary rule, especially when the small open economy experiences foreign shocks. The differences arise because with the exchange rate rule there is no overshooting of the nominal exchange rate which helps stabilize terms of trades without significantly increasing the volatility of monetary policy targets such as inflation and output.

These results are confirmed for our baseline calibrations by the second moments of key macroeconomic variables reported in Table (A.2, columns 1-2).

4.2.2 Third-Order: Deviations from Uncovered Interest Parity

We now show that differences in business cycle dynamics between the two rules are amplified at a third-order approximation, due to the existence of a time-varying risk premium. The time-varying risk premium reflects fluctuations in precautionary saving motive which may stabilize or amplify fluctuations in monetary policy targets such as inflation.

As argued before, after a positive domestic productivity shock, consumption increases under both rules, however more so under the IRR. Since consumption prospects improve upon impact – consumption is hump-shaped or decreases only slowely – domestic households become less precautionary, save less and consume more (Figure B.1 versus B.3). Hence, the presence of an endogenous risk premium amplifies fluctuations in consumption. The risk premium on foreign bonds is counter-cyclical and decreases, more so under the IRR where the exchange rate is more volatile.

Under the IRR, where interest rates are fixed by the central bank, lower precautionary savings lead to more capital outflows, stronger immediate depreciation (overshooting) and a larger expected appreciation. Thus, the lower risk premium offers an improvement in terms of trade and rebalances aggregate demand to domestic goods. Prices would need to decrease less in order to clear a given excess supply for home goods. However, higher consumption means also a higher marginal utility of working and output raises as well. In total, prices need to decrease more. Under the ERR, on the other hand, lower precautionary savings imply higher domestic interest rates. For a given path of current and expected deflation, households save more and consume less (inter-temporal substitution effect). Since the risk premium is very stable under the ERR, the additional price decrease is small.

After a positive foreign output shock (see Figure B.4) foreign and domestic consumption go up. Since the foreign shock is sufficiently persistence, both domestic and foreign households become less precautionary, save less and consume even more. However since foreign consumption increases more, precautionary savings decrease more in the rest of the world leading to capital inflows under both rules. The risk premium is pro-cyclical and increases, more so under IRR where the exchange rate is more volatile.

Under the IRR, the additional capital inflows (due to less precautionary foreign investors) leads to an additional nominal appreciation and amplifies the deterioration in terms of trade. Demand rebalances away from home towards foreign goods. Prices have to decrease more, and the central bank sets interest rates even lower to stimulate demand. The risk premium dynamics thus destabilize the small open economy. Under the ERR, on the other hand, the higher consumption (due to less precautionary savings) increases the marginal utility of working. Output increases in response to the foreign output shock such that excess demand decreases and prices increase by less. The central bank under the ERR appreciates the currency by less, which further stabilizes the nominal exchange rate. The risk premium dynamics thus stabilize the small open economy under the ERR.

Our qualitative results show that the two monetary policy rules generate different business cycle dynamics when UIP holds (see also Table A.2, columns 4-5). These differences are amplified by deviations from UIP. The exchange rate rule generates less fluctuations than the Taylor rule, especially when the economy is exposed to foreign shocks. That is, small open economies that are exposed to shocks originating in the rest of the world may benefit from rules that use the exchange rate to stabilize the economy.

4.3 Generalized Monetary Rules

In this section, we analyze the performance of the different rules in terms of second moments. We augment the rules from section 4.2 along the following dimensions: *(i)* we allow the monetary authority to react to deviations of inflation and output (Figure B.5); *(ii)* we allow for the IRR to react also to the exchange rate (Figure B.6); *(iii)* we compare the ERR with one in which the monetary authority leaves the exchange rate fixed, which we call the PEG. In all of the cases, we evaluate the rules under a wide range of plausible values for the parameters.

The particular functional forms of the monetary rules is as follows:

$$\hat{r}_t = \rho\,\hat{r}_{t-1} + (1-\rho)\left[\phi_\Pi \hat{\pi}_t + \phi_Y \hat{y}_t + \phi_E \Delta\hat{e}_t\right] \tag{IRR}$$

$$\Delta\hat{e}_t = \rho\,\Delta\hat{e}_{t-1} - (1-\rho)\left[\phi_\Pi \hat{\pi}_t + \phi_Y \hat{y}_t\right] \tag{ERR}$$

$$\Delta\hat{e}_t = 0 \tag{PEG}$$

with $\rho = 0.85$, $\phi_\Pi \leq 5$, [26] and $\phi_Y \in [0, 1]$. Figure B.5 shows the volatility of key economic variables for various values of ϕ_Y and ϕ_Π under our rules. In this comparison, we assume $\phi_E = 0$. Figure B.6 shows the volatility of key economic variables for various values of ϕ_Y and ϕ_Π under the ERR and an IRR in which we allow the central bank to react also to fluctuations of the nominal exchange rate. We consider two cases: *(i)* $\phi_E = 0.2$ and *(ii)* $\phi_E = 0.8$. We do the analysis for the hybrid model. In this case, the different performance between the two rules will be driven by the interaction between the implementation of the rule and the existence of a time-varying risk premium.

In Figure B.5, we find that both the ERR and the peg outperform the IRR in terms of reducing fluctuations of all variables variables except for consumption growth and the domestic interest rate. These differences are larger the lower the response to inflation and the larger the response to output deviations. Under the ERR, the monetary authority is more effective at stabilizing the economy by introducing less fluctuations in the risk premium. When comparing the ERR with the peg, we find that the peg does better than the ERR for smoothing domestic interest rates and exchange rates, it seems to perform similarly for output and consumption growth but does worse in terms of inflation and domestic inflation. As the parameters of the rule increase in value, the differences between the peg and the ERR are more striking. Under the peg, the central bank's objective is to keep the exchange rate fixed, and this sometimes comes at the expense of generating larger fluctuations in other parts of the economy. With the ERR, however, the central bank manages to smooth fluctuations of the exchange rate by allowing for a large degree of smoothing of the instrument, while still being able to react to fluctuations of inflation and the output gap. This is especially the case when the monetary authority that follows an ERR puts a strong weight on fluctuations of inflation and output.

In Figure B.6, we consider the same rules as before, but we allow the IRR to react, in addition to deviations of inflation and output, to fluctuations of exchange rates through the parameter ϕ_E. We consider two values for this parameter: *(i)* a low and conservative value of $\phi_E = 0.2$ and *(ii)* a stronger and less conservative value of $\phi_e = 0.8$. We find that the differences between the ERR and the IRR are similar when $\phi_E = 0$ or when $\phi_E = 0.2$. We start observing larger differences when the reaction to exchange rate fluctuations is larger, i.e. when $\phi_E = 0.8$. In that case, the IRR performs very similarly or slightly

[26] Several previous studies on optimal monetary policy tend to restrict $\phi_\Pi \leq 3$ (see Schmitt-Grohé & Uribe (2007)).

better than the ERR in terms of smoothing fluctuations of domestic inflation and CPI inflation when ϕ_π is large. For low values of ϕ_π, however, the performance of the IRR in terms of smoothing economic fluctuations is much worse than that of the ERR. That is, if the central bank puts a big weight both on smoothing the volatility of the exchange rate and inflation, the IRR would perform very similarly to the ERR in terms of second moments.

4.4 Welfare Analysis

We compare the performance of the IRR and the ERR in terms of the implied lifetime utility of the household for a wide range of parameter combinations in the two rules (in all the exercises we keep the smoothness parameter fixed at $\rho = 0.85$). We compute welfare by taking a third order approximation of the full model and of the utility function. In that way, we capture variations in the risk premium of the economy and we are able to evaluate how they impact the preferences of the central bank. We consider the same generalized monetary policy rules as above. The results are reported in Figure B.7. We analyze the differences between the rules in terms of welfare for: *(i)* our baseline economy; *(ii)* an economy with a higher elasticity of substitution; and *(iii)* an economy with a higher degree of openness. The shaded gray area represents ranges of parameter values for ϕ_Π and ϕ_Y for which the ERR generates larger welfare – in terms of life-time utility – than the IRR.

The first row of Figure B.7 plots the results for our baseline model and for three different values of ϕ_E in the IRR. When we do not allow the central bank to react to fluctuation of the exchange rate in the IRR, the ERR outperforms in terms of welfare as long as the reaction of ϕ_Π is not too large. For values of ϕ_Π above 3, the IRR generates larger welfare. As the reaction to exchange rate fluctuations increases (second and third columns in the first row of the Figure), the IRR starts outperforming the ERR in terms of welfare for a wider set of parameters.

The second row of Figure B.7 plots the results for a version of our model that has a larger elasticity of substitution between domestic and foreign goods (i.e. $\eta = 2$). In this case, the ERR outperforms the IRR in terms of welfare for every plausible combination of parameter values, when $\phi_E = 0$ and when $\phi_E = 0.2$. When the reaction to exchange rates fluctuations is stronger, that is when $\phi_E = 0.8$, for large values of ϕ_π and ϕ_y, the IRR generates larger welfare.

Finally, the third row of Figure B.7 plots the results for a version of our model with a higher degree of openness (i.e. $\alpha = 0.4$). In this case the ERR outperforms the IRR in terms of welfare for every plausible combination of parameter values of ϕ_π, ϕ_y and ϕ_e. That is, as the economy becomes more open, following an exchange rate rule yields better results in terms of welfare than following a standard IRR, regardless whether or not we allow the the central bank to react to fluctuations of exchange rates.

In conclusion, a policy rule in which the central bank adjusts the exchange rate to react to fluctuations of output and inflation is welfare improving with respect to a monetary rule in which the central bank uses the interest rate as its instrument. This is especially the case for very open economies and for economies in which the elasticity of substitution between domestic and foreign goods is large.[27]

5 Conclusion

We have shown how a theoretical model based on optimizing behavior of households and producers is able to generate powerful conclusions about the desirability to implement monetary policy through flexible exchange rate targeting. More generally, we show that in a standard microfounded monetary model, relatively open economies can stabilize the economy better through exchange rate rules rather than the traditional rules based on interest rate adjustments.

The model reveals that there are two key sources of the reduction in macroeconomic fluctuations. First, in implementing the exchange rate rule, the central bank announces a gradual depreciation rate, which avoids the standard overshooting result. For small open economies, where a large part of the price level is determined by prices of imported goods, this policy already reduces the volatility of exchange rates and thus prices of imports. To identify the second channel, we follow recent advances in international monetary economics and asset pricing, which build into standard models counter-cyclical risk premia derived endogenously from habits in consumption. The time-varying risk premia drive a wedge between exchange rate movements and the interest rate differential thus further separating the implied dynamics of interest rate rules from the dynamics implied by adopting an exchange rate rule. The time series properties of the risk premium differ considerably between the rules.

We have kept the model to a bare minimum in terms of its economic structure in order to identify the key factors behind the observed differences between the two rules. There are many directions in which the model can be extended to gain further insights in the desirability of exchange rate rules. First and foremost, increasing the interest rate sensitivity of key economic sectors may lead to a significant improvement in the performance of the interest rate rule. This may occur due to the importance of investment in economic fluctuations or by including a financial accelerator as in Bernanke, Gertler & Gilchrist (1999). Second, our model does not distinguish between tradable and non-tradable goods. At the same time, secular changes in relative prices due to convergence in

[27]These results are robust to variations of the rules in which: (i) both rules react to deviations of domestic inflation instead of CPI inflation; (iii) both rules react to output growth rather than deviations of output from its steady-state level. The ERR performs better than the peg for all parameter combinations.

income per capita, for example, might present interesting problems for the exchange rate rule. Finally, in the past few years, standard monetary policy rules have been put to a test by the well-known zero lower bound on nominal interest rates. This minimum bound created a problem for economies that use the interest rate as an instrument of monetary policy, forcing them to switch to quantitative easing once interest rates reached zero. For an economy operating with an exchange rate rule, the challenge is different (Amador, Bianchi, Bocola & Perri (2017)). When the anchor currency country lowers rates to zero, while the domestic economy overheats, the response should be future appreciation of the domestic currency. To meet the no-arbitrage condition implied by UIP, domestic rates have to go below zero. And even though in the past few years negative rates have been observed, there is a limit to how low negative rates can go. These three extensions are not only interesting from a modeling point of view, but they are clearly relevant for the actual implementation of the exchange rate rule in small open economies.

Finally, Singapore is not the only country that has used the exchange rate to stabilize the economy of monetary policy. From 6 September 2011 to 15 January 2015, the Swiss National Bank (SNB) introduced as its key monetary policy instrument the minimum exchange rate of CHF 1.20 per euro. The main goal was to correct the massive overvaluation of the Swiss franc. The Czech National Bank (CNB) decided in November 2013 to start using the exchange rate as an additional instrument for easing the monetary conditions, after having lowered the interest rate to almost zero. The CNB intervened in the foreign exchange market to weaken the koruna so as to achieve the desired easing of the monetary conditions as mandated by the Bank Board. In many ways, Switzerland and the Czech Republic share similar characteristics to the Singapore economy. They are small open economies with a long-term liquidity in the banking sector. It is not surprising that these economies monitor closely the exchange rate. What distinguishes them from just following the exchange rate is that they have explicitly used it as an instrument of monetary policy to achieve their monetary policy mandates. The particular implementation of the exchange rate rules in those countries differs from the one in Singapore. While both Switzerland and the Czech Republic have abandoned the use of exchange rate rules to stabilize their economies, Singapore continued using it since 1980, and it has been quite successful at stabilizing inflation and output. Even prior to the successful introduction of the exchange rate as instrument of monetary policy in Singapore, there were experiments with exchange-rate based monetary policy in Latin America (the so-called Southern Cone Stabilization Plans). To fight against high inflation, Argentina, Chile and Uruguay introduced in the late 1970s pre-announced schedules of depreciation for the exchange rate (tablitas). The tablitas were active crawling pegs, where the central bank pre-announced the future values of the nominal exchange rate over a specified horizon. The announcements were expected to directly lower the prices of traded goods and also to lower inflationary expectations and thus future prices.

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Appendix

A Tables

Table A.1: Baseline Calibration of the Model's Parameters

This table shows the calibration of the model's parameters used in all baseline simulations if not explicitely specified differently.

Parameter		Value	Source
External Habits in Consumption			
h	Degree of External Consumption Habits	0.85	GM2005
δ	Persistence of External Consumption Habits	0.99	DePS2009
Household Preferences			
β	Time-Discount Factor	0.99	GM2005
σ	Inverse of the Elasticity of Intertemporal Substitution	5	DePS2009
γ	Inverse of the Frisch Labor Supply Elasticity	$3/(1-h)$	DePS2009
ε	Elasticity of Substitution between Varieties	6	GM2005
η	Trade Elasticity	1	GM2005
α	Degree of Openness	0.08	DePS2009
Production Technology			
θ	Calvo Price Stickiness	0.67	GM2005
Exogenous Processes			
σ_A	Std. Deviation of Domestic Productivity Shock	0.0071	GM2005
ρ_A	Persistence of Domestic Productivity	0.66	GM2005
σ_{Y^*}	Std. Deviation of Foreign Output Shock	0.0078	GM2005
ρ_{Y^*}	Persistence of Domestic Productivity	0.86	GM2005
σ_{A,Y^*}	Corr. btw. Domestic Prod. and Foreign Output Shock	0.3	GM2005

Note: GM2005: Gali & Monacelli (2005), DePS2009: De Paoli & Sondergaard (2009)

Table A.2: Moments under Simplified Monetary Policy Rules

This table shows simulated moments of selected variables of the model under an inflation targeting interest rate rule ($\phi_\pi = 1.5$, IRR), exchange rate rule ($\phi_\pi = 1$, ERR), and a fixed exchange rate regime (PEG). Baseline calibrations are used. The economy is subject to domestic productivity and foreign output shocks. Simulations are performed in the log-linearized model (Log-Linear, columns 2-4) and in the model where all equations are approximated to the third order, with the exception of demand and supply equations (Hybrid, columns 5-7). Simulations are done in Dynare, each model is simulated for 10,000 periods, and for the hybrid model the pruning option is used.

	Log-Linear			***Hybrid***		
	IRR	*ERR*	*PEG*	*IRR*	*ERR*	*PEG*
Real Growth Rates: Standard Deviations						
Output, ΔY_t	1.92%	0.49%	0.47%	1.64%	0.61%	0.60%
Consumption, ΔC_t	0.42%	0.70%	0.70%	0.48%	0.76%	0.76%
Real Depreciation Rate, ΔQ_t	13.88%	4.61%	4.54%	12.33%	2.65%	2.60%
Nominal Growth Rates: Standard Deviations						
Domestic Inflation, $\Pi_{H,t}$	7.77%	4.37%	4.93%	9.26%	2.52%	2.83%
CPI Inflation, Π_t	8.65%	3.98%	4.54%	9.95%	2.29%	2.60%
Domestic Interest Rate, R_t	4.86%	8.10%	7.66%	5.79%	7.57%	7.33%
Foreign Interest Rate, R_t^*	7.66%	7.66%	7.66%	7.33%	7.33%	7.33%
Nominal Depreciation Rate, ΔE_t	21.21%	1.04%	0.00%	20.44%	0.60%	0.00%
Interest Rate Differential, $R_t - R_t^*$	2.99%	0.89%	0.00%	2.03%	0.52%	0.00%
FX Risk Premium: Moments						
Mean	0.00	0.00	0.00	-0.012	-0.001	0.00
Standard Deviation	0.00%	0.00%	0.00%	0.16%	0.01%	0.00%

B Figures

Figure B.1: Responses to a Domestic Productivity Shock - Log-Linear Approximation

This figure shows the responses of key variables (in %-deviation from steady state) to a positive domestic productivity shock of one percent implied by a fixed exchange rate regime (PEG), an interest rate (IRR), and an exchange rate rule (ERR) targeting CPI inflation only. The model is approximated to the first order.

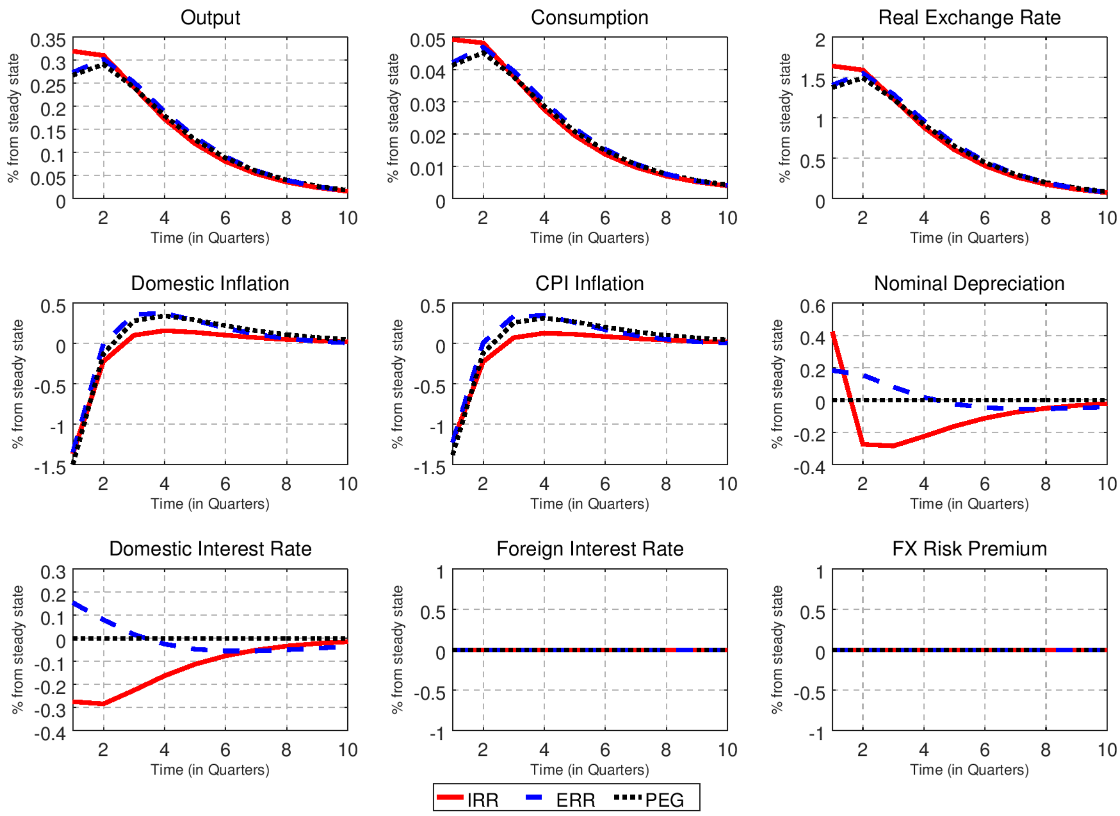


Figure B.2: Responses to a Foreign Output Shock - Log-Linear Approximation

This figure shows the responses of key variables (in %-deviation from steady state) to a positive foreign output shock of one percent implied by a fixed exchange rate regime (PEG), an interest rate (IRR), and an exchange rate rule (ERR) targeting CPI inflation only. The model is approximated to the first order.

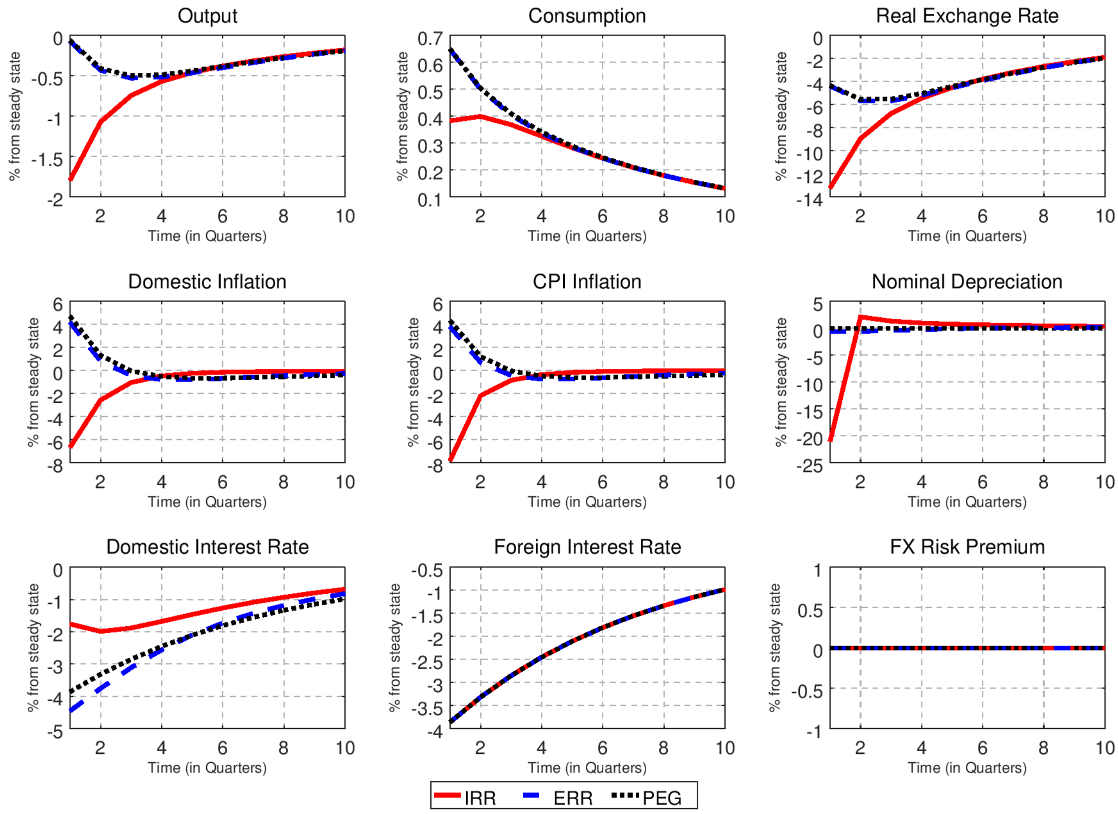


Figure B.3: Responses to a Domestic Productivity Shock - 3rd Order Approximation

This figure shows the responses of key variables (in %-deviation from steady state) to a positive domestic productivity shock of one percent implied by a fixed exchange rate regime (PEG), an interest rate (IRR), and an exchange rate rule (ERR) targeting CPI inflation only. The model is approximated to the 3rd order.

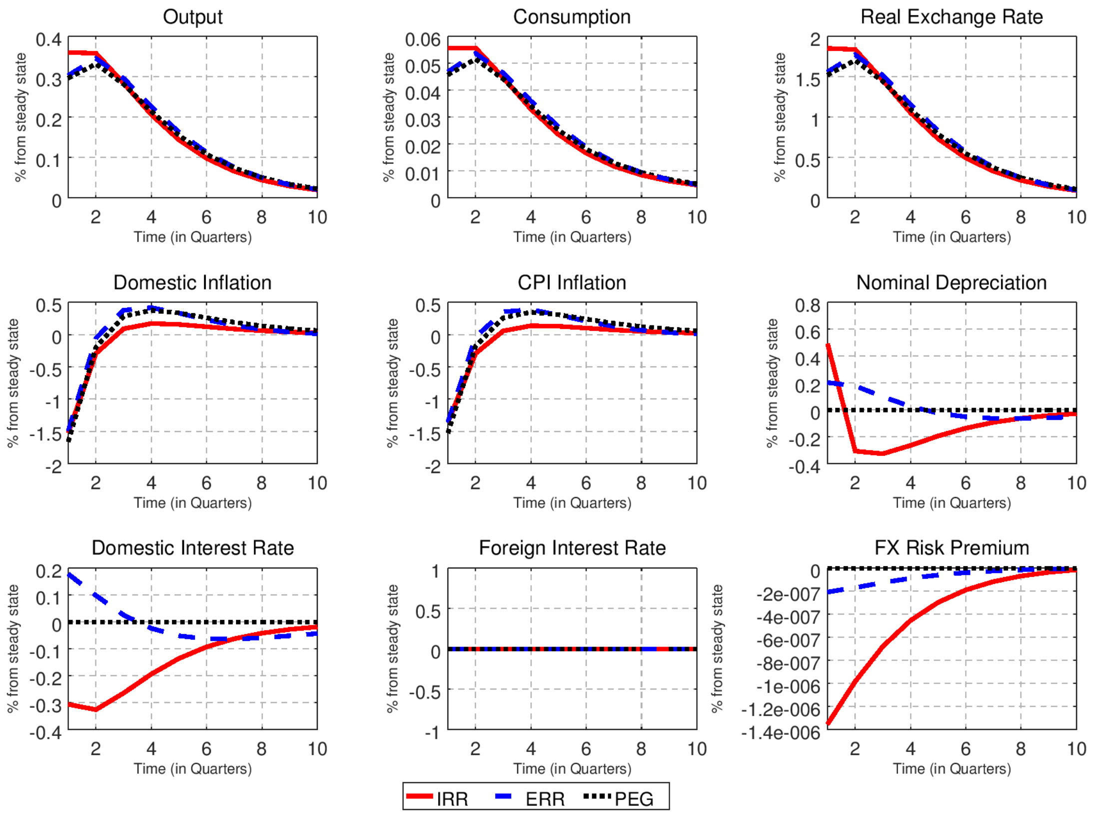


Figure B.4: Responses to a Foreign Output Shock - 3rd Order Approximation

This figure shows the responses of key variables (in %-deviation from steady state) to a positive foreign output shock of one percent implied by a fixed exchange rate regime (PEG), an interest rate (IRR), and an exchange rate rule (ERR) targeting CPI inflation only. The model is approximated to the 3rd order.

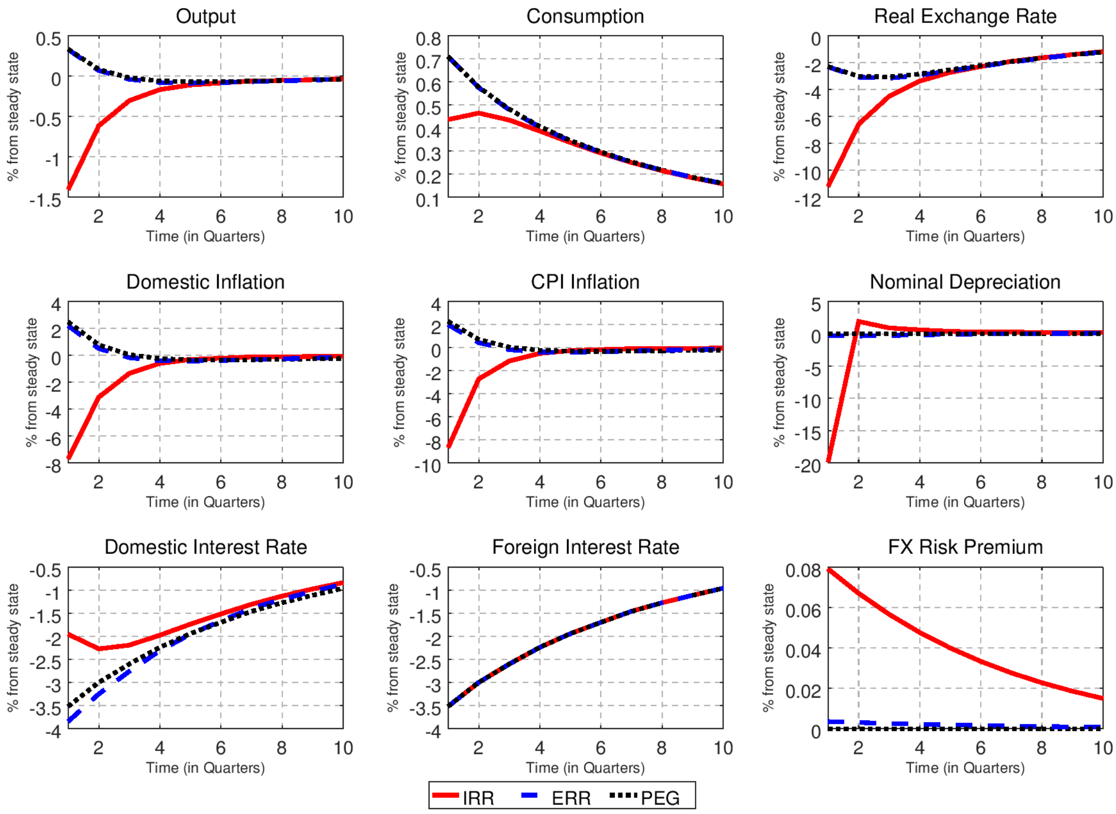


Figure B.5: Volatilities under Augmented Rules

This figure shows the standard deviations (in %) of key variables implied by a fixed exchange rate regime (PEG), and interest rate (IRR) and exchange rate rules (ERR) targeting CPI inflation (ϕ_{Π}) and output (ϕ_{Y}). The model is approximated at the third order with the exception of the supply and demand equations. Simulations are obtained in Dynare for 10,000 periods.

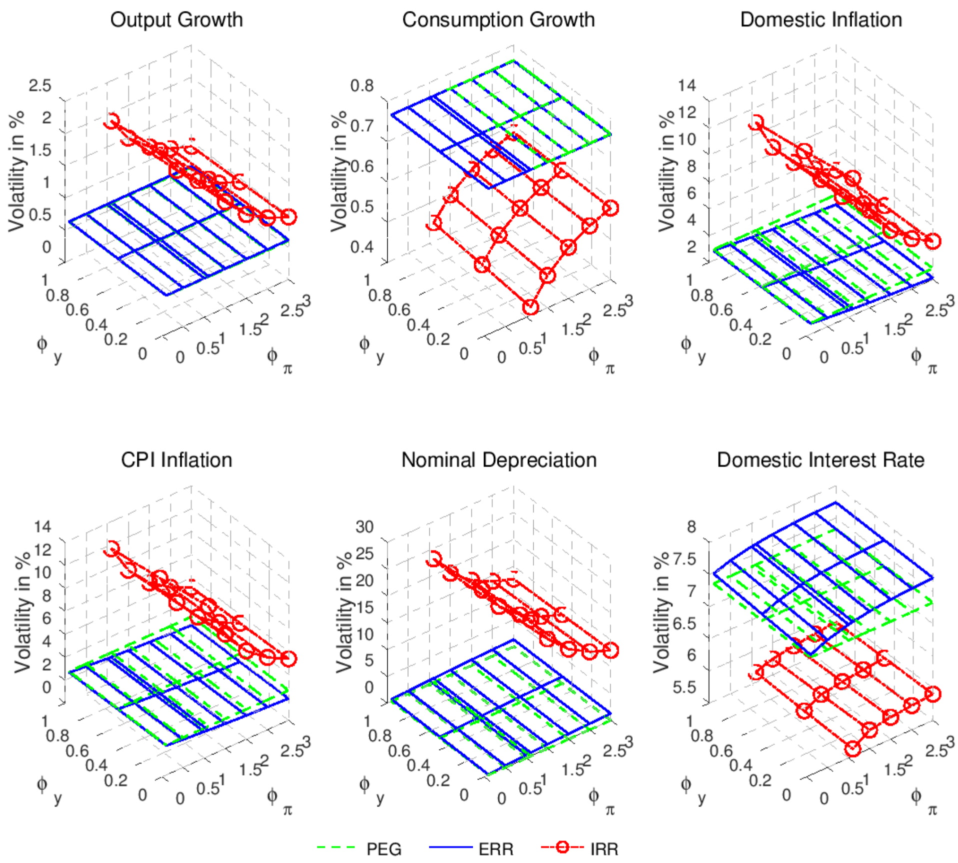


Figure B.6: Volatilities under Augmented Rules - IRR with Exchange Rate Target

This figure shows the standard deviations (in %) of key variables implied by an exchange rate rule (ERR) and two interest rate (IRR) rules. In addition to CPI inflation (ϕ_Π) and output (ϕ_Y), the interest rate rules also target the nominal depreciation, one with high intensity ($\phi_E = 0.8$) and the other with low intensity ($\phi_E = 0.2$). The model is approximated at the third order with the exception of the supply and demand equations. Simulations are obtained in Dynare for 10,000 periods.

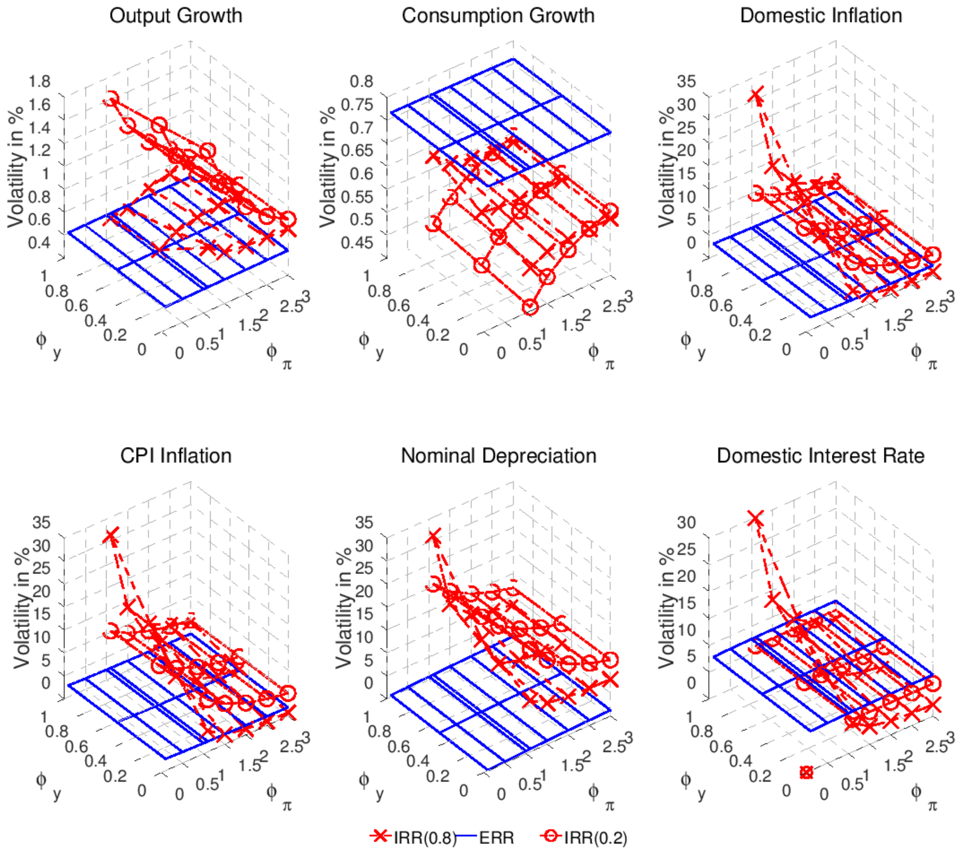
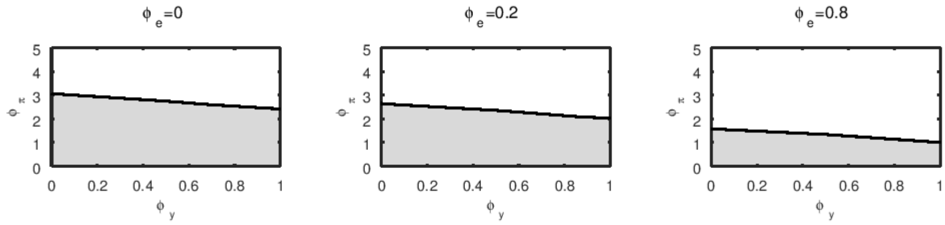


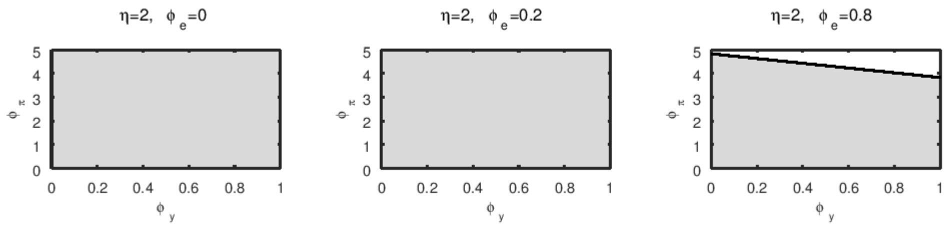
Figure B.7: Monetary Poliy Rule Parameters ϕ_π and ϕ_y and Welfare

This figure shows the space of monetary policy rule parameters, ϕ_π and ϕ_y, for which the exchange rate rule (ERR) outperforms interest rate rules (IRR) with no ($\phi_e = 0$, column 1), weak ($\phi_e = 0.2$, column 2) and strong ($\phi_e = 0.8$, column 3) nominal depreciation targeting. Rules are compared for the baseline calibration (Panel (a)) and the robustness scenarios of high trade elasticity (Panel (b)), and high openness (Panel (c)). The grey shaded area depicts the parameter space for which the ERR outperforms the IRR.

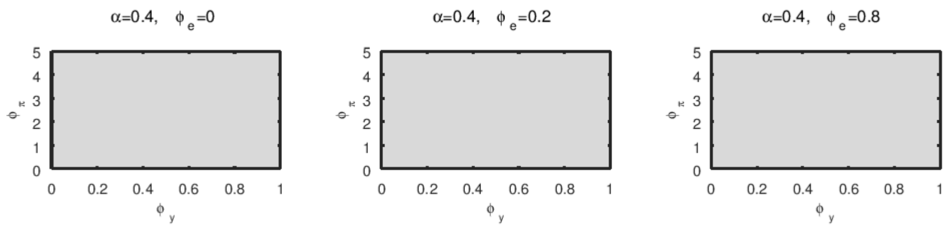
(a) Baseline Calibration



(b) High Trade Elasticity



(c) High Openness



C Online Appendix: The Model's Equilibrium Equations

The paths of endogenous variables,

$$\left\{C_t, X_t, C_{Ht}, C_{Ft}, A_t, W_t, \frac{P_{Ht}}{P_t}, \frac{P_{Ft}}{P_t}, \Pi_t, N_t, R_t, S_t, Q_t, C_t^*, e_t, Y_t, N_t, MC_t, Y_t^*, \Pi_t^*, R_t^*\right\},$$

are determined by the following equilibrium conditions.

Households

$$R_t E_t \left\{ \beta \left(\frac{C_{t+1} - hX_{t+1}}{C_t - hX_t} \right)^{-\sigma} \frac{1}{\Pi_{t+1}} \right\} = 1$$

$$X_t = \delta X_{t-1} + (1-\delta) C_{t-1}$$

$$C_{H,t} = (1-\alpha) \left(\frac{P_{H,t}}{P_t} \right)^{-\eta} C_t$$

$$C_{F,t} = \alpha \left(\frac{P_{F,t}}{P_t} \right)^{-\eta} C_t$$

$$(C_t - hX_t)^{\sigma} N_t^{\gamma} = \frac{W_t}{P_t}$$

Firms

$$Y_t = A_t N_t$$

$$a_t = \rho a_{t-1} + u_t$$

$$W_t = MC_t P_{Ht} A_t$$

Price Setting

$$\frac{\tilde{P}_{H,t}}{P_t} = \frac{\varepsilon}{\varepsilon - 1} \frac{H_t}{F_t}$$

$$F_t = \Lambda_t Y_t + \theta \beta E_t (F_{t+1} (\Pi_{t+1})^{\varepsilon - 1})$$

$$H_t = \Lambda_t MC_t Y_t + \beta \theta E_t (H_{t+1} (\Pi_{t+1})^{\varepsilon})$$

$$\Lambda_t = (C_t - hX_t)^{-\sigma}$$

$$\frac{P_{Ht}}{P_t} = \left((1-\theta) \left(\frac{\tilde{P}_{H,t}}{P_t} \right)^{1-\varepsilon} + \theta \left(\frac{P_{Ht-1}}{P_{t-1}} \right)^{1-\varepsilon} \Pi_t^{\varepsilon - 1} \right)^{\frac{1}{1-\varepsilon}}$$

Goods Market Clearing

$$Y_t = C_t \left[(1-\alpha) S_t^{\eta} Q_t^{-\eta} + \alpha Q_t^{-\frac{1}{\sigma}} \right]$$

Monetary Policy Rule

$$\frac{R_t}{\bar{R}} = \left(\frac{R_{t-1}}{\bar{R}} \right)^{\rho} \left(\frac{Y_t}{\bar{Y}} \right)^{(1-\rho)\phi_y} \left(\frac{\Pi_t}{\bar{\Pi}} \right)^{(1-\rho)\phi_m}$$

$$\frac{e_{t+1}}{e_t} = \left(\frac{e_{t+1}^*}{e_t^*} \right)^{(1-\rho_e)} \left(\frac{e_t}{e_{t-1}} \right)^{\rho_e}$$

Terms of Trade and the Real Exchange Rate

$$S_t = \frac{P_{Ft}}{P_{Ht}}$$

$$Q_t = \frac{P_{Ft}}{P_t}$$

$$P_{Ft} = \mathcal{E}_t P_t^*$$

Uncovered Interest Parity

$$E_t \left\{ \mathcal{M}_{t,t+1} \left(R_t - R_t^* \frac{\mathcal{E}_{t+1}}{\mathcal{E}_t} \right) \right\} = 0$$

Domestic Euler Equation

$$\beta \left(\frac{C_{t+1} - hX_{t+1}}{C_t - hX_t} \right)^{-\sigma} \left(\frac{P_t}{P_{t+1}} \right) = \mathcal{M}_{t,t+1}$$

Rest of the World

$$C_t^* = Y_t^*$$

$$\left(\frac{Y_t^*}{\bar{Y}} \right) = \left(\frac{Y_{t-1}^*}{\bar{Y}} \right)^{\rho_y} exp(u_t^y)$$

$$R_t^* E_t \left\{ \beta \left(\frac{C_{t+1}^* - hX_{t+1}^*}{C_t^* - hX_t^*} \right)^{-\sigma} \right\} = 1$$

We solve for the symmetric steady state.

$$
\begin{aligned}
\tilde{C} &= (1-h)C \\
X &= C \\
\beta R &= 1 \\
P_H &= P \\
P_F &= P \\
C_H &= (1-\alpha)C \\
C_F &= \alpha C \\
Y &= C \\
\frac{W}{P} &= \omega = \frac{\varepsilon-1}{\varepsilon} \\
Y &= N \\
N &= (1-h)^{\frac{-\sigma}{\sigma+\psi}}\left(\frac{\varepsilon-1}{\varepsilon}\right)^{\frac{1}{\sigma+\psi}} \\
Q &= 1 \\
MC &= \frac{\varepsilon-1}{\varepsilon} \\
F &= \frac{\Lambda Y}{1-\theta\beta} \\
H &= F\cdot MC \\
\Lambda &= C^{-\sigma}(1-h)^{-\sigma} \\
\frac{\tilde{P}_H}{P_t} &= 1
\end{aligned}
$$

D Online Appendix: Derivation of ERR Transmission Channels

This appendix derives Equations (33, 34) illustrating the mechansim.

Intra-Temporal Demand Rebalancing Channel We start by log-linearizing demand for the home good (Equation 27),

$$\hat{y}_t^d = -\eta\hat{p}_{H,t} + (1-\alpha)\hat{c}_t + \eta\alpha\hat{q}_t + \alpha\hat{c}_t^*, \tag{D.1}$$

and replace the log-deviation of the domestic price (in real terms), $\hat{p}_{H,t}$, with

$$\hat{p}_{H,t} = -\frac{\alpha}{1-\alpha}\hat{q}_t, \tag{D.2}$$

from the log-linearization of the CPI index. This yields

$$\hat{y}_t^d = \frac{\eta\alpha(2-\alpha)}{1-\alpha}\hat{q}_t + (1-\alpha)\hat{c}_t + \alpha\hat{c}_t^*. \tag{D.3}$$

Using the log-linear definition of the real exchange rate, $\hat{q}_t = \Delta\hat{e}_t - \hat{\pi}_t + \hat{q}_{t-1}$, we can replace the log-deviation of the nominal depreciation rate with the simplified exchange rate rule (Equation 32) and obtain

$$\hat{y}_t^d = \frac{\eta\alpha(2-\alpha)}{1-\alpha}\left(\rho\Delta\hat{e}_{t-1} - (1+\rho(1-\phi_\Pi)\hat{\pi}_t) + \hat{q}_{t-1}\right) + (1-\alpha)\hat{c}_t + \alpha\hat{c^*}_t. \tag{D.4}$$

Total differentiation holding $t-1$-variables constant yields Equation (33) in the text.

Inter-Temporal Consumption-Saving Channel To illustrate the inter-temporal consumption-saving channel, we log-linearize the domestic Euler Equation (Equation 12),

$$\hat{\hat{c}}_t = -\sigma^{-1}\left(\hat{r}_t - \mathbb{E}_t\left[\hat{\pi}_{t+1}\right]\right) + \mathbb{E}_t\left[\hat{\hat{c}}_{t+1}\right], \tag{D.5}$$

and use the uncovered interest parity condition,

$$\hat{r}_t^* = \hat{r}_t + \mathbb{E}_t\left[-\Delta\hat{e}_{t+1}\right], \tag{D.6}$$

to see how foreign interest rates and currency appreciations transmit to domestic consumption choice,

$$\hat{\hat{c}}_t = -\sigma^{-1}\left(\hat{r}_t^* + \mathbb{E}_t\left[\Delta\hat{e}_{t+1}\right] - \mathbb{E}_t\left[\hat{\pi}_{t+1}\right]\right) + \mathbb{E}_t\left[\hat{\hat{c}}_{t+1}\right]. \tag{D.7}$$

Together with the simplified exchange rate rule (Equation 32), we obtain

$$\widehat{\hat{c}_t} = -\sigma^{-1}\left(\widehat{r_t^*} + \rho^2\widehat{\Delta e_{t-1}} - \rho(1-\rho)\phi_\Pi\widehat{\pi_t} - (1+(1-\rho)\phi_\Pi)\mathbb{E}_t\left[\widehat{\pi_{t+1}}\right]\right) + \mathbb{E}_t\left[\widehat{\hat{c}_{t+1}}\right]. \tag{D.8}$$

Total differentation holding $t-1$-variables constant and solving the Euler equation forward gives

$$\begin{aligned} d\hat{\hat{c}}_t &= -\sigma^{-1}\mathbb{E}_t\left[\sum_{i=0}^{\infty} d\hat{r}_{t+i}^* - \rho(1-\rho)\phi_\Pi\sum_{i=0}^{\infty} d\hat{\pi}_{t+i} - (1+(1-\rho)\phi_\Pi)\sum_{i=0}^{\infty} d\hat{\pi}_{t+1+i}\right] \\ &\quad + \lim_{i\to\infty}\mathbb{E}_t\left[d\hat{\hat{c}}_{t+1+i}\right]. \end{aligned} \tag{D.9}$$

Since we consider non-explosive solutions only, $\lim_{i\to\infty}\mathbb{E}_t\left[d\hat{\hat{c}}_{t+1+i}\right] = 0$. Gali & Monacelli (2005) show that under an exchange rate peg prices are stationary. We verify numerically that the ERR also implies stationary prices, which, in turn, implies that the sum of all inflation deviations from steady state is zero, $\sum_{i=0}^{\infty} d\hat{\pi}_{t+i} = 0$. Addition and substraction

$-(1+(1-\rho)\phi_\Pi)d\hat{\pi}_t$ gives

$$d\hat{\tilde{c}}_t = -\sigma^{-1}(1+(1-\rho)\phi_\Pi)d\hat{\pi}_t - \sigma^{-1}\sum_{i=0}^{\infty}\mathbb{E}_t\left[d\hat{r}^*_{t+i}\right]. \tag{D.10}$$

Equivalently, solving forward the foreign Euler Equation,

$$d\hat{\tilde{c}}^*_t = -\sigma^{-1}\sum_{i=0}^{\infty}\mathbb{E}_t\left[d\hat{r}^*_{t+i}\right], \tag{D.11}$$

and using the log-linearization of habit-adjusted consumption, $\hat{\tilde{c}}_t = (1-h)^{-1}(\hat{c}_t - h\hat{x}_t)$, allows to solve for consumption,

$$d\hat{c}_t = -(1-h)\sigma^{-1}(1+(1-\rho)\phi_\Pi)\cdot d\hat{\pi}_t + d\hat{c}^*_t, \tag{D.12}$$

where $d\hat{x}_t = \delta d\hat{x}_{t-1} + (1-\delta)\hat{c}_{t-1} = 0 = d\hat{x}^*_t$ holding $t-1$-variables constant. This is Equation (34) in the text.

E Online Appendix: Welfare-Based Loss-Function and Optimal Policy

We derive, analytically, a welfare-based loss function of the small open economy described above. In this model, the decentralized equilibrium allocation generally does not coincide with the efficient allocation. This is because of four features of the model that distort the decentralized equilibrium away from efficiency: price stickiness à la Calvo, monopolistic competition between producers of different varieties, a terms of trade externality and external habits (Corsetti & Pesenti (2001), Faia & Monacelli (2008), Leith, Moldovan & Rossi (2012)). We follow closely De Paoli (2009) who uses the linear-quadratic approach developed by Sutherland (2002) and Benigno & Woodford (2006). De Paoli (2009) shows that the welfare cost of the decentralized equilibrium of the model can be accurately expressed by a purely quadratic loss function of the policy maker in so-called welfare gaps, i.e. deviations of a variable from the Ramsey planner's equilibrium. In our model there are welfare gaps in four variables, each representing one distortion: domestic inflation, domestic production, the real exchange rate and the relative risk aversion. This loss function takes the following form

$$\mathbb{L}_0 \;=\; U'_{\tilde{C}}\tilde{C}\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\left[\frac{1}{2}\Phi_Y\tilde{y}_t^2 + \frac{1}{2}\Phi_\Pi\tilde{\pi}_H^2 + \frac{1}{2}\Phi_Q\tilde{q}_t^2 + \frac{1}{2}\Phi_{RA}\tilde{ra}_t^2\right] + t.i.p. + \mathcal{O}(||\epsilon||^3)$$

where $t.i.p.$ stands for terms independent of policy, $\tilde{y}_t = y_t - y_t^T$, $\tilde{\pi}_{H,t} = \pi_{H,t} - 0$, $\tilde{q}_t = q_t - q_t^T$ and $\tilde{ra}_t = ra_t - ra_t^T$ are welfare gaps in domestic production, domestic inflation, the real exchange rate and the relative risk aversion, respectively. In this model, the relative risk aversion is defined as

$$RA_t \equiv \frac{\sigma C_t}{C_t - hX_t}$$

where ρ is the coefficient of intertemporal elasticity of substitution; h is the habit parameter; and $X_t = (1-\delta)C_{t-1} + \delta X_{t-1}$, with δ the parameter of persistence of habit in consumption. Consumers choose their consumption, taking X_t as given, so that habits are external.

Each Φ_i determine the relative importance of welfare gaps for the loss and are function of structural parameters of the model. The gaps in the loss function are deviations of the decentralized economy from the efficient allocation. It is important to note that, for the purpose of our exercise, we do not introduce policy instruments that that allow the Ramsey planner to achieve the first best (i.e. close all welfare-gaps). In particular, there is no employment subsidy that eliminates the mark-up. This is in contrast to De Paoli (2009) and other papers that derive the loss function analytically.

The optimal policy path A benevolent Ramsey planner minimizes the loss function subject to the private sector's optimal behavior by choosing the optimal paths of the model's variables given exogenous process of domestic productivity and foreign output shocks.[28] We are imposing time-consistency. The Ramsey planner minimizes its loss at the beginning of time taking initial conditions as given and commits to the first order conditions of this problem (see Woodford (2001)). The resulting rule is called *optimal targeting rule*, which implements the optimal policy from a welfare point of view. This policy is also sometimes called *Ramsey policy.* The problem of the Ramsey planner can be written as follows:

$$\min_{\pi_{H,t},\tilde{ra}_t,\tilde{y}_t,\tilde{q}_t} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{1}{2}\Phi_Y \tilde{y}_t^2 + \frac{1}{2}\Phi_Q \tilde{q}_t^2 + \frac{1}{2}\Phi_{RA}\tilde{ra}_t^2 + \frac{1}{2}\Phi_\Pi \pi_H^2\right] + t.i.p. + \mathcal{O}(||\epsilon||^3)$$

$$\begin{array}{rrcll} & \pi_{H,t} & = & \lambda\left(\gamma\tilde{y}_t + \frac{1}{1-\alpha}\tilde{q}_t + \hat{u}_t\right) + \beta\mathbb{E}_t[\tilde{\pi}_{H,t+1}] & (\mu_{1,t}) \\ s.t. & \tilde{y}_t & = & \frac{l+1}{\sigma(1-\alpha)}\tilde{q}_t + (1-\alpha)\tilde{ra}_t + \hat{o}_t & (\mu_{2,t}) \\ & \tilde{ra}_t & = & [\delta(1-h)+h]\tilde{ra}_{t-1} - \frac{h}{\sigma}(\tilde{q}_t - \tilde{q}_{t-1}) + \hat{d}_t & (\mu_{3,t}) \end{array}$$

[28] Note that the optimal monetary policy problem can also be solved in a non-linear way (see Faia & Monacelli (2008)). The advantage of the linear-quadratic approach implemented here, is comparability with more ad-hoc loss functions and intuition on objectives and mechanisms conveyed through it.

The solution to the policy maker's optimization problem can be obtained by minimizing the associated Lagrangian. The first order conditions are:

$$\begin{aligned}
\Phi_Y \tilde{y}_t &= -\lambda\gamma\mu_{1,t} + \mu_{2,t} \\
\Phi_Q \tilde{q}_t + \frac{\lambda}{1-\alpha}\mu_{1,t} + \frac{1+l}{\sigma(1-\alpha)}\mu_{2,t} &= \frac{h}{\sigma}\mu_{3,t} - \beta\frac{h}{\sigma}\mu_{3,t+1} \\
\Phi_{RA}\tilde{ra}_t + (1-\alpha)\mu_{2,t} &= -\beta(\delta(1-h)+h)\mu_{3,t+1} + \mu_{3,t} \\
\Phi_\Pi \pi_{H,t} &= \mu_{1,t} - \mu_{1,t-1}
\end{aligned}$$

Let Δ denote the first difference operator between t and $t-1$. We can re-arrange:

$$\begin{aligned}
\sigma(1-\alpha)\Phi_Q\Delta\tilde{q}_t + \lambda(\sigma+(1+l)\gamma)\Phi_\Pi\pi_{H,t} + (l+1)\Phi_Y\Delta\tilde{y}_t &= (1-\alpha)h(\Delta\mu_{3,t} - \beta\Delta\mu_{3,t+1}) \\
\delta_{RA}\Delta\tilde{ra}_t + (1-\alpha)\Phi_Y\Delta\tilde{y}_t + \lambda\gamma(1-\alpha)\Phi_\Pi\pi_{H,t} &= \Delta\mu_{3,t} - \beta[\delta(1-h)+h]\Delta\mu_{3,t+1}
\end{aligned}$$

Now we solve for $\Delta\mu_{3,t}$ and $\Delta\mu_{3,t+1}$.[29] The optimal policy plan is

$$\Phi'_q\tilde{q}_t + \hat{\Phi}_q\mathbb{E}[\tilde{q}_{t+1}] + \Phi'_a\tilde{ra}_t - \hat{\Phi}_a\mathbb{E}[\Delta\tilde{ra}_{t+1}] + \Phi'_Y\Delta\tilde{y}_t + \hat{\Phi_Y}\mathbb{E}[\Delta\tilde{y}_{t+1}] + \Phi'_\Pi\pi_{H,t} + \hat{\Phi}_\Pi\mathbb{E}[\pi_{H,t+1}]$$

where the weights on the optimal rule are a function of the weights in the loss function in equation (E.1) and the deep parameters of the model.[30]. We find that, when $h = 0$, the targeting rule is identical to that in De Paoli (2009) and the term related to the coefficient of relative risk aversion disappears. The derivation of the loss function follows closely De Paoli (2009). The difference is that here we introduce habit formation in Home and Foreign. The loss function then depends not only on the welfare-gaps in output, real exchange rate and domestic inflation, but also risk aversion. In the following the real price of the domestic good is $p_{H,t} \equiv \frac{P_{H,t}}{P_t}$, RA_t is constant relative risk aversion. Risk aversion is time varying because of habits in consumption. Define the following vectors:

$$y_t = \begin{pmatrix} \hat{Y}_t \\ \hat{C}_t \\ \hat{p_{H,t}} \\ \hat{Q}_t \\ \hat{RA}_t \end{pmatrix}, \quad e_t = \begin{pmatrix} \hat{A}_t \\ \hat{C}^*_t \\ \hat{RA}^*_t \end{pmatrix}$$

[29] These derivations are done in *Mathematica*.

[30] In particular, $\Phi'_Q = -\Phi_Q(-1+\alpha)\sigma\Delta$, $\hat{\Phi}_q = -\Phi_Q(-1+\alpha)\beta(h(-1+\gamma)-\gamma)\sigma$, $\Phi'_a = h\Phi_{RA}(-1+\alpha)\Delta$, $\hat{\Phi}_a = h\Phi_{RA}(-1+\alpha)\beta$, $\Phi'_Y = \Phi_Y\left(1+l-h(-1+\alpha)^2\right)$, $\hat{\Phi_Y} = \Phi_Y\beta(-(1+l)\gamma + h((-2+\alpha)\alpha + l(-1+\gamma)+\gamma))$, $\Phi'_\Pi = \Phi_\Pi\lambda\left(\left(1+l-h(-1+\alpha)^2\right)\delta+\sigma\right)$ and $\hat{\Phi}_\Pi = \Phi_\Pi\beta\lambda(h((-2+\alpha)\alpha + l(-1+\gamma)+\gamma)\delta + h(-1+\gamma)\sigma - \gamma(\delta+l\delta+\sigma))$

E.1 Second-Order Approximation of Life-Time Utility

$$W_0 = U'_{\tilde{C}}\tilde{C}\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[w'_y y_t - \frac{1}{2} y'_t W_y y_t - y'_t W_e e_t - \frac{1}{2} w_\pi \pi^2_{H,t} \right] + t.i.p. + \mathcal{O}(||\epsilon||^3)$$

$$w_y = \begin{pmatrix} -\frac{1}{\mu'} \\ 0 \\ 0 \\ \frac{1}{\sigma} \\ 0 \end{pmatrix}, \quad w_\pi = \frac{\epsilon}{\kappa\mu'}, \quad \mu' = \mu(1-h) = \frac{\epsilon(1-\tau)}{\epsilon - 1}(1-h)$$

$$-W_y = \begin{pmatrix} \frac{(1+\gamma)}{\mu'} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{(1-\sigma)}{\sigma^2} & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \quad -W_e = \begin{pmatrix} -\frac{(1+\gamma)}{\mu'} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & -\frac{(1-\sigma)}{\sigma} & \frac{(1-\sigma)}{\sigma} \\ 0 & 0 & 0 \end{pmatrix}$$

E.2 Second-Order Approximation of Equilibrium Conditions

We aim to express utility in second-order terms only and therefore need to substitute the linear terms in the utility approximation by second-order terms using the model's equilibrium conditions. We need to approximate the following equilibrium conditions.

Price Setting	$\frac{\bar{P}_{H,t}}{P_t} = p_{H,t}\frac{\epsilon}{\epsilon-1}\frac{\tilde{C}_t^{-\sigma}MC_tY_t+\beta\theta\mathbb{E}_t\left[H_{t+1}\Pi_{t+1}^{-1}\Pi_{H,t+1}^{\epsilon+1}\right]}{\tilde{C}_t^{-\sigma}Y_t+\beta\theta\mathbb{E}_t\left[F_{t+1}\Pi_{t+1}^{-1}\Pi_{H,t+1}^{\epsilon}\right]}$
Market Clearing	$Y_t = p_{H,t}^{-\eta}\left[(1-\alpha)C_t + \alpha Q_t^{\eta}C_t^*\right]$
Real Exchange Rate	$(1-\alpha)p_{H,t}^{1-\eta} = 1 - \alpha Q_t^{1-\eta}$
Risk Sharing	$\frac{\sigma C_t}{RA_t} = Q_t^{\frac{1}{\sigma}}\frac{\sigma C_t^*}{RA_t^*}$
LoM Risk Aversion	$\frac{\sigma C_t}{RA_t} = C_t - [(1-\delta)h+\delta]C_{t-1} + \delta\frac{\sigma C_{t-1}}{RA_{t-1}}$

E.2.1 Price Setting

The derivation is very similar to the one in the technical appendix to Benigno and Benigno (2003).

$$V_{t_0} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[a'_y y_t + \frac{1}{2} y'_t A_y y_t + y'_t A_e e_t + \frac{1}{2} a_\pi \pi^2_{H,t} \right] + t.i.p. + \mathcal{O}(||\epsilon||^3)$$

$$a_y = \begin{pmatrix} \gamma \\ 0 \\ -1 \\ 1 \\ 0 \end{pmatrix}, \; A_y = \begin{pmatrix} \gamma(2+\gamma) & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & -1 & 1 & 0 \\ 1 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}, A_e = \begin{pmatrix} -(1+\gamma)^2 & \sigma & -\sigma \\ 0 & 0 & 0 \\ 0 & \sigma & -\sigma \\ 0 & -\sigma & \sigma \\ 0 & 0 & 0 \end{pmatrix}$$

$$a_\pi = \epsilon \tfrac{1+\gamma}{\kappa}, \quad \kappa = \tfrac{(1-\theta\beta)(1-\theta)}{\theta}$$

E.2.2 Market Clearing

The first order approximation of real exchange rate equilibrium condition is used to replace for $\hat{p}^2_{H,t}$.

$$\mathbb{E}_0 \textstyle\sum_{t=0}^{\infty} \beta^t \left[d_y' y_t + \frac{1}{2} y_t' D_y y_t + y_t' D_e e_t \right] + t.i.p. + \mathcal{O}(||\epsilon||^3) = 0$$

$$d_y = \begin{pmatrix} -1 \\ (1-\alpha) \\ -\eta \\ \eta\alpha \\ 0 \end{pmatrix}, \; D_y = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & \alpha(1-\alpha) & 0 & -(1-\alpha)\alpha\eta & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & -(1-\alpha)\alpha\eta & 0 & \alpha(1-\alpha)\eta^2 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}, D_e = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -(1-\alpha)\alpha & 0 \\ 0 & 0 & 0 \\ 0 & (1-\alpha)\alpha\eta & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

E.2.3 Real Exchange Rate

$$\mathbb{E}_0 \textstyle\sum_{t=0}^{\infty} \beta^t \left[f_y' y_t + \frac{1}{2} y_t' F_y y_t + y_t' F_e e_t \right] + \mathcal{O}(||\epsilon||^3) = 0$$

$$f_y = \begin{pmatrix} 0 \\ 0 \\ -(1-\alpha) \\ -\alpha \\ 0 \end{pmatrix}, \quad F_y = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\alpha(\eta-1) & 0 \\ 0 & 0 & -\alpha(\eta-1) & \frac{\alpha(\eta-1)(1-2\alpha)}{1-\alpha} & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}, F_e = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

E.2.4 International Risk Sharing

This equation is exact log-linear. Second-order terms are thus equal to zero.

$$c_y' y_t + c_e' e_t = 0, \quad c_y = \begin{pmatrix} 0 \\ 1 \\ 0 \\ -\frac{1}{\sigma} \\ -1 \end{pmatrix}, \quad c_e = \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}$$

E.2.5 Law of Motion for Risk Aversion

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[z_y' y_t + \tfrac{1}{2} y_t' Z_y y_t + y_t' Z_e e_t \right] + t.i.p. + \mathcal{O}(||\epsilon||^3) = 0$$

$$z_y = \begin{pmatrix} 0 \\ \frac{h(1-k)}{1-h} \\ 0 \\ 0 \\ 1 \end{pmatrix}, \quad Z_y = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{h(1-k)}{1-h} & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 \end{pmatrix}, Z_e = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$k = \frac{(1-\delta)\beta}{1-\delta\beta}$$

E.3 Replacing Linear Terms in Utility Approximation

E.3.1 Derivation of the Vector Lx

First we need to find the linear combination of linear terms in equilibrium conditions that equals linear terms in the utility approximation. Let Lx denote this vector:

$$\begin{pmatrix} a_y & d_y & f_y & c_y & z_y \end{pmatrix} Lx = w_y$$

$$\begin{pmatrix} \gamma & -1 & 0 & 0 & 0 \\ 0 & (1-\alpha) & 0 & 1 & \frac{h(1-k)}{1-h} \\ -1 & -\eta & -(1-\alpha) & 0 & 0 \\ 1 & \eta\alpha & -\alpha & -\frac{1}{\sigma} & 0 \\ 0 & 0 & 0 & -1 & 1 \end{pmatrix} Lx \quad = \quad \begin{pmatrix} -\frac{1}{\mu'} \\ 0 \\ 0 \\ \frac{1}{\sigma} \\ 0 \end{pmatrix}$$

The solution to this linear equation system is found in *Mathematica*.

E.3.2 Second-Order Loss Function

We use L_x to weight the second-order terms of the equilibrium conditions.

$$\begin{aligned} L_y &= W_y + L_{x_1} A_y + L_{x_2} D_y + L_{x_3} F_y + L_{x_5} Z_y \\ L_e &= W_e + L_{x_1} A_e + L_{x_2} D_e \\ l_\pi &= w_\pi + L_{x_1} a_\pi \end{aligned}$$

Now the loss function writes as

$$\mathbb{L}_0 = U'_{\tilde{C}} \tilde{C} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{1}{2} y'_t L_y y_t + \frac{1}{2} l_\pi \pi^2_{H,t} + y'_t L_e e_t \right] + t.i.p. + \mathcal{O}(||\epsilon||^3)$$

E.3.3 Mapping into $\{\hat{Y}_t, \hat{Q}_t, \hat{RA}_t\}$-Space

We use first first order approximations to goodmarket clearing, real exchange rate and risk sharing to replace $\hat{C}_t$ and $\hat{p}_{H,t}$:

$$\begin{pmatrix} \hat{Y}_t \\ \hat{C}_t \\ \hat{p_{H,t}} \\ \hat{Q}_t \\ \hat{RA}_t \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 1 & -\frac{l+\alpha}{\sigma(1-\alpha)} & \alpha \\ 0 & -\frac{\alpha}{1-\alpha} & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \hat{Y}_t \\ \hat{Q}_t \\ \hat{RA}_t \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -\alpha \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \hat{A}_t \\ \hat{C}^*_t \\ \hat{RA}^*_t \end{pmatrix} + \mathcal{O}(||\epsilon||^2)$$

We define $\tilde{y}'_t = (\hat{Y}_t,\, \hat{Q}_t,\, \hat{RA}_t)$ and

$$L'_y = \begin{pmatrix} a & b & c \\ b & d & e \\ c & e & f \end{pmatrix} \qquad L'_e = \begin{pmatrix} g & h & i \\ 0 & k & l \\ 0 & n & o \end{pmatrix}$$

then the loss function writes as:

$$\mathbb{L}_0 = U'_{\tilde{C}} \tilde{C} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{1}{2} \tilde{y}'_t L'_y \tilde{y}_t + \frac{1}{2} l_\pi \pi^2_{H,t} + \tilde{y}'_t L'_e e_t \right] + t.i.p. + \mathcal{O}(||\epsilon||^3)$$

E.3.4 Representation in Welfare Gaps

With the notation as above we can write the loss function as

$$\mathbb{L}_0 = U'_{\tilde{C}}\tilde{C}\frac{1}{2}\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\begin{bmatrix} a\hat{Y}_t^2 + d\hat{Q}_t^2 + f\hat{RA}_t^2 + l_\pi\pi_{H,t}^2 \\ 2b\hat{Y}_t\hat{Q}_t + 2c\hat{Y}_t\hat{RA}_t + 2e\hat{RA}_t\hat{Q}_t \\ +2\hat{Y}_t(g\hat{A}_t + h\hat{C}_t^* + i\hat{RA}_t^*) \\ +2\hat{Q}_t(k\hat{C}_t^* + l\hat{RA}_t^*) \\ +2\hat{RA}_t(n\hat{C}_t^* + o\hat{RA}_t^*) \\ +l_\pi\pi_{H,t}^2 \end{bmatrix} + t.i.p. + \mathcal{O}(||\epsilon||^3)$$

To write the loss function in terms of gaps from a policy target (=functions of exogenoues shocks only) in $\hat{Y}_t$, $\hat{Q}_t$, $\hat{RA}_t$ and $\pi_{H,t}$, we use

$$\hat{Y}_t = \frac{l+1}{\sigma(1-\alpha)}\hat{Q}_t + (1-\alpha)\hat{RA}_t + \hat{C}_t^* - (1-\alpha)\hat{RA}_t^*$$

from first order approximations to market clearing, real exchange rate and risk sharing. With this we replace interaction terms as follows

$$\begin{aligned}
2b\hat{Y}_t\hat{Q}_t &= b\Big[\tfrac{\sigma(1-\alpha)}{l+1}\hat{Y}_t^2 + \tfrac{l+1}{\sigma(1-\alpha)}\hat{Q}_t^2 - \tfrac{\sigma(1-\alpha)^3}{l+1}\hat{RA}_t^2 - \tfrac{\sigma(1-\alpha)}{l+1}\hat{Y}_t\left(\hat{C}_t^* - (1-\alpha)\hat{RA}_t^*\right) \\
&+ \quad \hat{Q}_t\left(\hat{C}_t^* - (1-\alpha)\hat{RA}_t^*\right) - \tfrac{\sigma(1-\alpha)^2}{l+1}\hat{RA}_t\left(\hat{C}_t^* - (1-\alpha)\hat{RA}_t^*\right)\Big] \\
2c\hat{Y}_t\hat{RA}_t &= c\Big[\tfrac{1}{1-\alpha}\hat{Y}_t^2 - \tfrac{(l+1)^2}{\sigma^2(1-\alpha)^3}\hat{Q}_t^2 + (1-\alpha)\hat{RA}_t^2 - \tfrac{1}{1-\alpha}\hat{Y}_t\left(\hat{C}_t^* - (1-\alpha)\hat{RA}_t^*\right) \\
&- \quad \tfrac{l+1}{\sigma(1-\alpha)^2}\hat{Q}_t\left(\hat{C}_t^* - (1-\alpha)\hat{RA}_t^*\right) + \hat{RA}_t\left(\hat{C}_t^* - (1-\alpha)\hat{RA}_t^*\right)\Big] \\
2e\hat{RA}_t\hat{Q}_t &= e\Big[\tfrac{\sigma}{l+1}\hat{Y}_t^2 - \tfrac{l+1}{\sigma(1-\alpha)^2}\hat{Q}_t^2 - \tfrac{\sigma(1-\alpha)^2}{l+1}\hat{RA}_t^2 - \tfrac{\sigma}{l+1}\hat{Y}_t\left(\hat{C}_t^* - (1-\alpha)\hat{RA}_t^*\right) \\
&- \quad \tfrac{1}{1-\alpha}\hat{Q}_t\left(\hat{C}_t^* - (1-\alpha)\hat{RA}_t^*\right) - \tfrac{\sigma(1-\alpha)}{l+1}\hat{RA}_t\left(\hat{C}_t^* - (1-\alpha)\hat{RA}_t^*\right)\Big]
\end{aligned}$$

Finally, we replace the interactions between endogenous variables using the above expressions and find the utility-based loss function in terms of welfare gaps only. The exact calculations are done in *mathematica*:

$$\begin{aligned}
\mathbb{L}_0 &= U'_{\tilde{C}}\tilde{C}\frac{1}{2}\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\begin{bmatrix}\Phi_Y(\hat{Y}_t - \hat{Y}_t^T)^2 + \Phi_Q(\hat{Q}_t - \hat{Q}_t^T)^2 \\ +\Phi_{RA}(\hat{RA}_t - \hat{RA}_t^T)^2 + \Phi_\Pi\pi_H^2\end{bmatrix} + t.i.p. + \mathcal{O}(||\epsilon||^3) \\
\mathbb{L}_0 &= U'_{\tilde{C}}\tilde{C}\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\left[\frac{1}{2}\Phi_Y\tilde{y}_t^2 + \frac{1}{2}\Phi_Q\tilde{q}_t^2 + \frac{1}{2}\Phi_{RA}\tilde{ra}_t^2 + \frac{1}{2}\Phi_\Pi\tilde{\pi}_H^2\right]
\end{aligned}$$